Activity Assess

2-8 Linear-Quadratic Systems

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I CAN… solve linear-quadratic systems.

MAFS.912.A-REI.3.7–Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. Also A-REI.4.11
MAFS.K12.MP.3.1, MP.4.1, MP.7.1

EXPLORE & REASON

Draw a rough sketch of a parabola and a line on the coordinate plane.

A. Count the number of points of intersection between the two graphs.

B. Sketch another parabola on a coordinate plane. Use a straightedge to investigate the different ways that a line and a parabola intersect. What conjectures can you make?

C. Construct Arguments How many different numbers of intersection points are possible between a quadratic function and a linear function? Justify that you have found all of the possibilities.

ESSENTIAL QUESTION

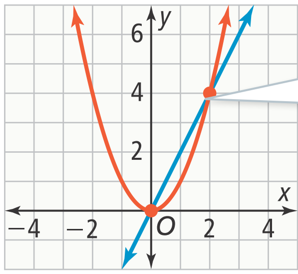
How can you solve a system of two equations or inequalities in which one is linear and one is quadratic?

CONCEPTUAL UNDERSTANDING

EXAMPLE 1 Determine the Number of Solutions

How many solutions can there be for a linear-quadratic system?

A. How many real solutions does the system $\begin{cases} y = x^2 \\ y = 2x \end{cases}$ have?

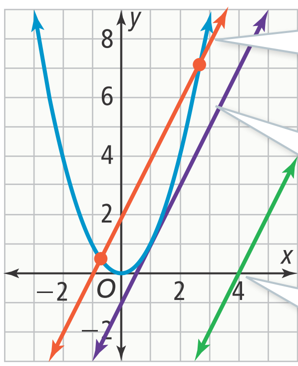


The graph seems to show that the quadratic function and linear function intersect at two points.

This system has two real solutions.

B. How does modifying the linear function in the system $\begin{cases} y = x^2 \\ y = 2x + b \end{cases}$ affect the number of solutions?

Test values for b to determine when the system has different numbers of solutions.



When $b = 2$, there are two solutions. The graphs intersect at two points.

When $b = -1$, there is one solution. The graph of $y = 2x - 1$ is tangent to the graph of $y = x^2$.

When $b = -8$, there are no solutions. The graph of $y = 2x - 8$ does not intersect the graph of $y = x^2$.

Visually inspecting the graph suggests that there is no way for a line to cross a parabola more than twice. Thus, the system of a linear function and a quadratic function may have 0, 1, or 2 solutions.

COMMUNICATE PRECISELY
A solution to a system of equations is an ordered pair that produces a true statement in all the equations of the system. In the graph, the solutions are the coordinates of the intersection points.

Try It! **1.** Determine the number of real solutions of the system $\begin{cases} y = 3x^2 \\ y = 3x - 2 \end{cases}$.

EXAMPLE 2 Solve a Linear-Quadratic System Using Substitution

USE STRUCTURE
As with a system of linear equations, you can use substitution and elimination to find the values of x and y that make the system true. In this case, substitution yields a new quadratic equation to solve.

How can you use substitution to solve this system? $\begin{cases} y = 3x^2 + 3x - 5 \\ 2x - y = 3 \end{cases}$

The first equation provides an expression for y in terms of x. Substitute this expression in the second equation.

$2x - (3x^2 + 3x - 5) = 3$ ······ Substitute $3x^2 + 3x - 5$ for y in the second equation.

$2x - 3x^2 - 3x + 5 = 3$ ······ Distribute −1 to remove parentheses.

$3x^2 + x - 2 = 0$ ······ Simplify.

$(x + 1)(3x - 2) = 0$ ······ Factor.

So $x = -1$ and $x = \frac{2}{3}$ are solutions of this quadratic equation.

If the graphs of the equations have two solutions, there are two points of intersection for the graphs of the equations.

When $x = -1$, $y = 2(-1) - 3$, or -5. When $x = \frac{2}{3}$, $y = 2\left(\frac{2}{3}\right) - 3$, or $-\frac{5}{3}$.

The solutions of the system are $(-1, -5)$ and $\left(\frac{2}{3}, -\frac{5}{3}\right)$.

Try It!

2. Solve each system by substitution.

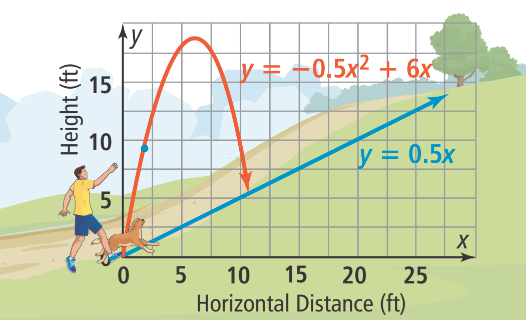
a. $\begin{cases} y = 2x^2 - 6x - 8 \\ 2x - y = 16 \end{cases}$

b. $\begin{cases} y = -3x^2 + x + 4 \\ 4x - y = 2 \end{cases}$

APPLICATION

EXAMPLE 3 Applying a Linear-Quadratic System

Andrew kicks a ball up a hill for his dog, Laika, to chase. The hill is modeled by a line through the origin. The path of the ball is modeled by the quadratic function shown. How far does the ball travel horizontally? How far must Laika run up the hill to catch it?



Create a system of equations and determine where the path of the ball intersects the hill.

$\begin{cases} y = -0.5x^2 + 6x \\ y = 0.5x \end{cases}$

$0.5x = -0.5x^2 + 6x$ ······ Substitute for y.

$0 = -0.5x^2 + 5.5x$ ······ Subtract $0.5x$ from both sides.

$0 = -0.5x(x - 11)$ ······ Factor.

$-0.5x = 0$ and $x - 11 = 0$ ······ Set each factor equal to 0 and solve.

$x = 0$ and $x = 11$

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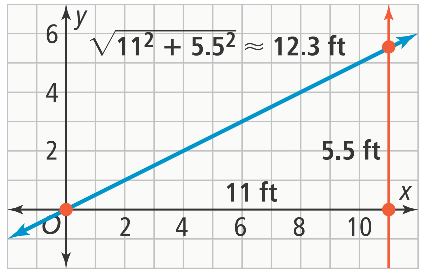
COMMON ERROR
Make sure to answer every part of the question. After solving this equation, you still need to find the distance that Laika ran.

EXAMPLE 3 CONTINUED

The solution $x = 0$ represents the horizontal distance, in feet, when Andrew kicks the ball. The solution $x = 11$ represents the horizontal distance, in feet, when the ball lands on the hill. So the ball travels 11 ft horizontally.



The route Laika runs can be modeled as the hypotenuse of a right triangle.

So Laika runs approximately 12.3 ft to get the ball.

Try It! **3.** Revenue for the high school band concert is given by the function $y = -30x^2 + 250x$, where x is the ticket price, in dollars. The cost of the concert is given by the function $y = 490 - 30x$. At what ticket price will the band make enough revenue to cover their costs?

EXAMPLE 4 Solve a Linear-Quadratic System of Inequalities

How can you solve this system of inequalities? $\begin{cases} y < -2x^2 + 12x - 10 \\ 4x + y > 4 \end{cases}$

Graphing an inequality is similar to graphing an equation. You start in the same manner, but later you have to consider whether to sketch the graph as a solid or dotted and how to shade the graph.

Graph the quadratic inequality:

Complete the square to write the inequality in vertex form.

$$y + 10 < -2(x^2 - 6x)$$
$$y + 10 - 18 < -2(x^2 - 6x + 9)$$
$$y - 8 < -2(x - 3)^2$$
$$y < -2(x - 3)^2 + 8$$

The parabola has vertex (3, 8).

Graph the linear inequality:

Solve the inequality for y to write in slope-intercept form:

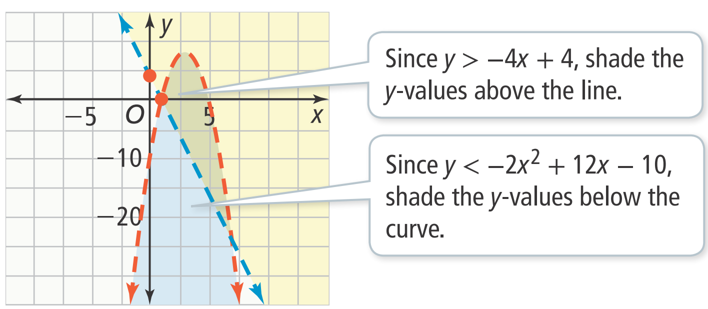
Sketch the graph of the linear inequality using the slope and y-intercept.

$$y > -4x + 4$$

Find two symmetric points on either side of the vertex:

$x = 2, y = 6 \rightarrow (2, 6)$ $x = 4, y = 6 \rightarrow (4, 6)$

Sketch the graph of the quadratic inequality using these three points.



The region where the two shaded areas overlap holds the solutions to the system.

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Try It! **4.** Solve the system of inequalities using shading. $\begin{cases} y > x^2 + 6x - 12 \\ 3x - y \geq -8 \end{cases}$

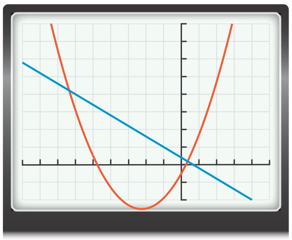
EXAMPLE 5 Using a System to Solve an Equation

STUDY TIP
If you graph both of these equations on your calculator, you can use the TRACE or INTERSECTION function to approximate the solution as a check.

Solve the equation $x^2 + 9x - 5 = 4 - 3x$ by writing a linear-quadratic system and using the intersection feature of a graphing calculator to solve it.

Graph the system $\begin{cases} y = x^2 + 9x - 5 \\ y = 4 - 3x \end{cases}$.

The graphing calculator shows the curve and line intersect at $x \approx 0.71$ and $x \approx -12.71$.



x scale: 2 *y scale: 10*

Check: Substitute the values into the original equation to verify the solutions. Because the values are approximations, one side of the equation should be approximately equal to the other.

$$x^2 + 9x - 5 = 4 - 3x$$
$$(0.71)^2 + 9(0.71) - 5 \stackrel{?}{=} 4 - 3(0.71)$$
$$0.5041 + 6.39 - 5 \stackrel{?}{=} 4 - 2.13$$
$$1.8941 \approx 1.87$$

$$x^2 + 9x - 5 = 4 - 3x$$
$$(-12.71)^2 + 9(-12.71) - 5 \stackrel{?}{=} 4 - 3(-12.71)$$
$$161.5441 - 114.39 - 5 \stackrel{?}{=} 4 + 38.13$$
$$42.1541 \approx 42.13$$

The equations show that the approximate solutions are reasonable.

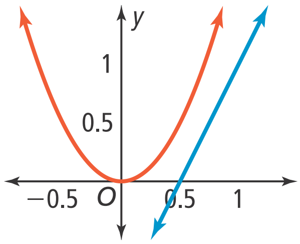
Try It! **5.** Solve the equation $3x^2 - 7x + 4 = 9 - 2x$ by writing a linear-quadratic system and solving using the intersection feature of a graphing calculator.

CONCEPT SUMMARY Key Features of Linear-Quadratic Systems

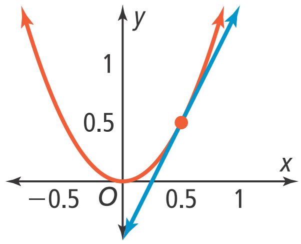
Linear-Quadratic Systems of Equations

WORDS Use substitution or elimination to solve the system.

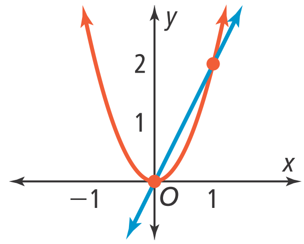
GRAPHS



no solution



1 solution

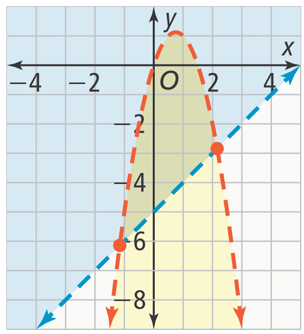


2 solutions

Linear-Quadratic Systems of Inequalities

WORDS Graph linear and quadratic inequalities, considering whether the graph is solid or dotted. Use shading to identify the solution region.

GRAPH



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you solve a system of two equations or inequalities in which one is linear and one is quadratic?

2. **Error Analysis** Dyani was asked to use substitution to solve this system:

$$\begin{cases} y = 2x^2 - 6x + 4 \\ x - y = 7 \end{cases}$$

She began as follows, to find the *x*-coordinate(s) to the solution(s) of the system:

$x + 2x^2 - 6x + 4 = 7$	Substitute for y.
$2x^2 - 5x - 3 = 0$	Simplify.
$(2x + 1)(x - 3) = 0$	Factor.
$x = -\frac{1}{2}, x = 3$	Set each factor equal to 0, solve for x. ✗

But Dyani has already made an error. What was her mistake?

Do You KNOW HOW?

Determine the number of solutions for the system of equations.

3. $\begin{cases} y = \frac{2}{5}x^2 \\ y = x - 2 \end{cases}$

4. $\begin{cases} y = -x - 1 \\ 3x^2 + 2y = 4 \end{cases}$

Use substitution to solve the system of equations.

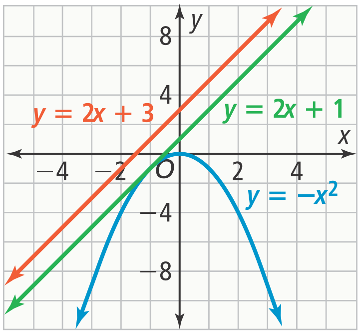
5. $\begin{cases} y = 3x^2 + 7x - 10 \\ y - 19x = 22 \end{cases}$

6. $\begin{cases} y = 3x^2 \\ y - 3x = -2 \end{cases}$

UNDERSTAND

7. **Construct Arguments** Nora and William are asked to solve the system of equations $\begin{cases} y - 1 = 3x \\ y = 2x^2 - 4x + 9 \end{cases}$ without graphing. Nora wants to use substitution, inserting $2x^2 - 4x + 9$ in place of y in the upper equation and solving. William wants to rewrite $y - 1 = 3x$ as $y = 3x + 1$ and begin by setting $3x + 1$ equal to $2x^2 - 4x + 9$, and then solving. Which student is correct, and why?

8. **Error Analysis** Chris was given the system of equations $\begin{cases} y = -x^2 \\ y = 2x + b \end{cases}$ and asked to use graphing to test the number of solutions of the system for different values of b. He graphed the system as shown, and concluded that the system could have one solution or no solutions depending on the value of b. What was Chris's error?



9. **Reason** You are given the following system of equations: $\begin{cases} y = x^2 \\ y = -1 \end{cases}$. Without graphing or performing any substitutions, can you see how many solutions the system must have? Describe your reasoning.

10. **Construct Arguments** Can a system of equations with one linear and one quadratic equation have more than two solutions? Give at least two arguments for your answer.

PRACTICE

Determine how many solutions each system of equations has by graphing them. SEE EXAMPLE 1

11. $\begin{cases} y = 3 \\ y = x^2 - 4x + 7 \end{cases}$

12. $\begin{cases} y = 3x^2 - 2x + 7 \\ y + 5 = \frac{1}{2}x \end{cases}$

Consider the system of equations $\begin{cases} y = x^2 \\ y = mx + b \end{cases}$.
SEE EXAMPLE 1

13. Find values for m and b so that the system has two solutions.

14. Find values for m and b so that the system has no solutions.

15. Find values for m and b so that the system has one solution.

Use substitution to solve the system of equations.
SEE EXAMPLE 2

16. $\begin{cases} y = 5 \\ y = 2x^2 - 16x + 29 \end{cases}$

17. $\begin{cases} y = 3x^2 - 4x \\ 27 + y = 14x \end{cases}$

18. LaToya throws a ball from the top of a bridge. Her throw is modeled by the equation $y = -0.5x^2 + 3x + 10$, and the bridge is modeled by the equation $y = -0.2x + 7$. About how far does the ball travel horizontally before its first bounce? SEE EXAMPLE 3

Solve each system of inequalities using shading.
SEE EXAMPLE 4

19. $\begin{cases} y > x^2 \\ 5 > y \end{cases}$

20. $\begin{cases} -5 < y - x \\ y < -3x^2 + 6x + 1 \end{cases}$

Solve each equation by writing a linear-quadratic system and solving using the intersection feature of a graphing calculator. SEE EXAMPLE 5

21. $6x^2 - 15x + 8 = 17 - 4x$

22. $7x^2 - 28x + 32 = 4$

23. $-\frac{5}{2}x - 10 = -2x^2 - x - 3$

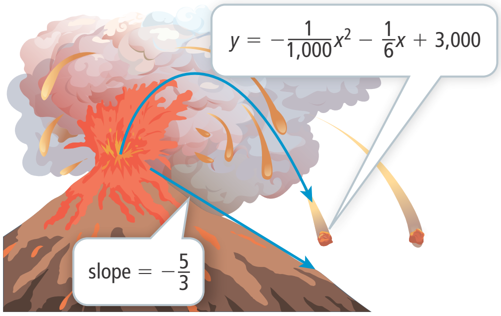
PRACTICE & PROBLEM SOLVING

Practice Tutorial

Mixed Review Available Online

APPLY

24. Model With Mathematics A boulder is flung out of the top of a 3,000 m tall volcano. The boulder's height, y, in meters, is a function of the horizontal distance it travels, x, in meters. The slope of the line representing the volcano's hillside is $-\frac{5}{3}$. At what height above the ground will the boulder strike the hillside? How far will it have traveled horizontally when it crashes?

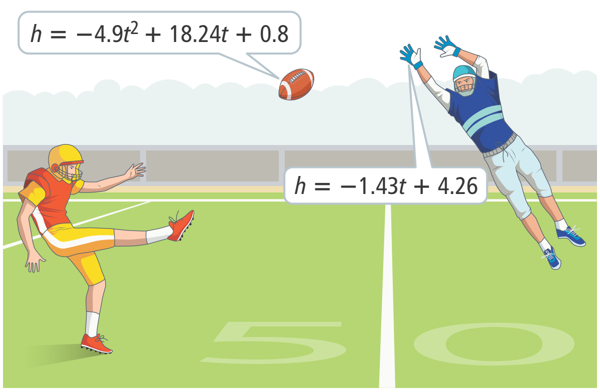


25. Use Structure You are given the system of equations:

$$\begin{cases} y = x + 1 \\ y^2 + x^2 = 25 \end{cases}$$

Solve the system using any of the methods you have learned in this lesson. Explain why you selected the method you used.

26. Reason A football player punts the football, whose path is modeled by the equation $h = -4.9t^2 + 18.24t + 0.8$ for h, in meters, and t, in seconds. The height of a blocker's hands for the same time, t, is modeled as $h = -1.43t + 4.26$. Is it possible for the blocker to knock down the ball? What else would you have to know to be sure?



ASSESSMENT PRACTICE

27. Select all functions that have exactly one point of intersection with the function $f(x) = x^2 + 8x + 11$. A-REI.3.7

☐ **A.** $g(x) = 2x - 12$

☐ **B.** $g(x) = 12x + 7$

☐ **C.** $g(x) = -5$

☐ **D.** $g(x) = 11 + 8x$

☐ **E.** $g(x) = -6$

28. SAT/ACT How many solutions does the following system of equations have?

$$\begin{cases} y = 16x - 19 \\ y = 3x^2 + 4x - 7 \end{cases}$$

Ⓐ two solutions

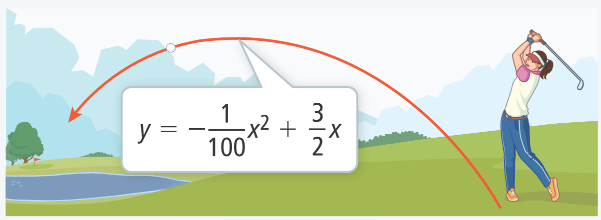
Ⓑ no solutions

Ⓒ an infinite number of solutions

Ⓓ one solution

Ⓔ The number of solutions cannot be determined.

29. Performance Task A golfer accidentally hits a ball toward a water hazard that is downhill from her current position on the fairway. The hill can be modeled by a line through the origin with slope $-\frac{1}{8}$. The path of the ball can be modeled by the function $y = -\frac{1}{100}x^2 + \frac{3}{2}x$.



Part A If the golfer stands at the origin, and the water hazard is 180 yd away, will the golfer's ball bounce or splash?

Part B How far did the ball land from the edge of the water hazard?

Part C Does it matter whether you measure the 180 yd horizontally or along the hill? Explain.