1-6

Linear Systems

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I CAN... use a variety of tools to solve systems of linear equations and inequalities.

VOCABULARY

• inconsistent system
• solution of a system of linear equations
• system of linear equations
• system of linear inequalities

MAFS.912.A-CED.1.3– Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret the solutions as viable or non-viable options in a modeling context. **Also A-CED.1.2, A-REI.3.6**

MAFS.K12.MP.4.1

CONCEPTUAL UNDERSTANDING

GENERALIZE
Recall that there are three possible outcomes when solving a system of two linear equations: no solution, one solution, or an infinite number of solutions.

EXPLORE & REASON

The graph shows two lines that intersect at one point.

A. What are the approximate coordinates of the point of intersection?

B. How could you verify whether the coordinates you estimated are, in fact, the solution? Is the point the solution to the equations of both lines?

C. Make Sense and Persevere Use your result to refine your approximation, and try again. Can you find the point of intersection this way? Is there a more efficient way?
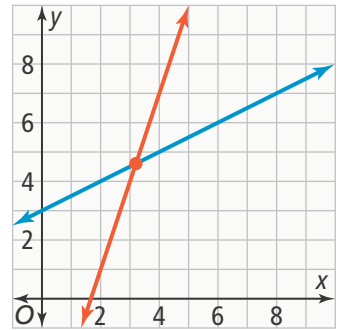

ESSENTIAL QUESTION

How can you find and represent solutions of systems of linear equations and inequalities?

EXAMPLE 1   Solve a System of Linear Equations

What is the solution of the system of linear equations $\begin{cases} x + 2y = 3 \\ x - 2y = 4 \end{cases}$?

A **system of linear equations** is a set of two or more equations using the same variables. The **solution of a system of linear equations** is the set of all ordered coordinates that simultaneously make all equations in the system true.

Sketch the graph of each equation to estimate the solutions. Then solve algebraically.
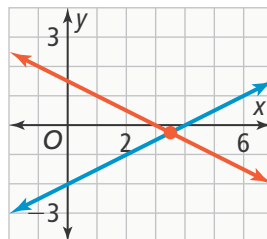
The x-coordinate of the solution is between 3 and 4, and the y-coordinate of the solution is between –1 and 0.

Substitute for x in both equations and solve.

Substitute the value for y into either original equation to find the value of x.

$x + 2y = 3 \;\Rightarrow\; x = 3 - 2y$
$x - 2y = 4 \;\Rightarrow\; x = 4 + 2y$

$\Downarrow$

$3 - 2y = 4 + 2y$
$-1 = 4y$
$-\dfrac{1}{4} = y$
$x = 3 - 2\left(-\dfrac{1}{4}\right)$
$x = \dfrac{7}{2}$

The solution is $\left(\dfrac{7}{2}, -\dfrac{1}{4}\right)$. These values are close to the estimate made from the graph. You can check to confirm that these values satisfy both equations.

☑ **Try It!**   **1.** Solve each system of equations.

a. $\begin{cases} 2x + y = -1 \\ 5y - 6x = 7 \end{cases}$

**b.** $\begin{cases} 3x + 2y = 5 \\ 6x + 4y = 3 \end{cases}$

APPLICATION   👆 **EXAMPLE 2**   Solve a System of Linear Inequalities

Malcolm earns $20 per hour mowing lawns and $10 per hour walking dogs. His goal is to earn at least $200 each week, but he can work a maximum of 20 h per week. Malcolm must spend at least 5 h per week walking his neighbors' dogs. For how many hours should Malcolm work at each job in order to meet his goals?

| $20 per hour | $10 per hour |
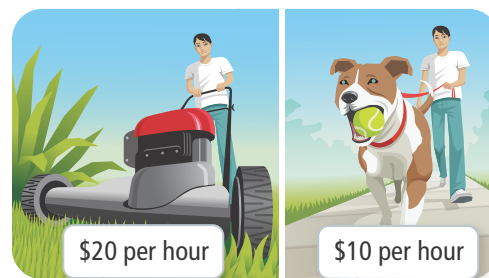
A **system of linear inequalities** is a set of two or more inequalities using the same variables.

Step 1 Define the variables.

x = number of hours spent mowing lawns

y = number of hours spent walking dogs

Step 2 Write inequalities to model the constraints.

Malcolm wants to earn at least $200 each week at $20 per hour mowing lawns and $10 per hour walking dogs: $20x + 10y \geq 200$.

Malcolm cannot work more than 20 h each week: $x + y \leq 20$.

Malcolm must spend at least 5 h walking dogs each week: $y \geq 5$.

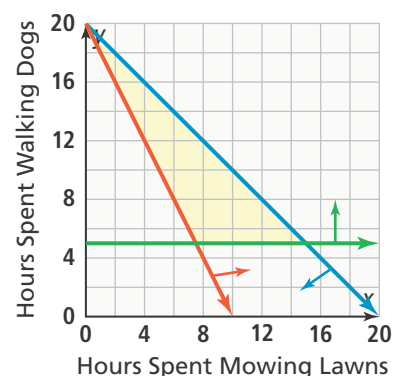**Step 3** Solve each inequality for y, then graph the inequalities on the same coordinate plane.

$y \geq 20 - 2x$

$y \leq 20 - x$

$y \geq 5$

Use arrows to show the region of the graph that satisfies each inequality.

Shade the region that satisfies all three inequalities.

Graph: Hours Spent Walking Dogs (y-axis, 0–20) vs. Hours Spent Mowing Lawns (x-axis, 0–20)

STUDY TIP
Using color to sketch graphs of equations or inequalities can make the graph easier to analyze.

Any point in the shaded region, such as (12, 7), is a solution to the system of inequalities. So if Malcolm spends 12 h mowing lawns and 7 h walking dogs, he will have met his goals.

✓ **Try It!**   **2.** Sketch the graph of the set of all points that solve this system of linear inequalities.

$$\begin{cases} 2x + y \leq 14 \\ x + 2y \leq 10 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

EXAMPLE 3   Solve a System of Equations in Three Variables

How can you solve a system of equations in three variables?

A. What is the solution of this system? $\begin{cases} 2x + y - z = -10 \\ -x + 2y + z = 3 \\ x + 2y + 3z = 13 \end{cases}$

$2x + y - z = -10$ (A)

$-x + 2y + z = 3$ (B)

$x + 2y + 3z = 13$ (C)

$x + 3y \quad\quad = -7$ (D)

$3x - 6y - 3z = -9$ (E)

$x + 2y + 3z = 13$ (F)

(D) = (A) + (B)

(E) = $-3 \times$ (B)

(F) = (C)

$x + 3y \quad\quad = -7$ (G)

$4x - 4y \quad\quad = 4$ (H)

$x + 2y + 3z = 13$ (J)

(G) = (D)

(H) = (E) + (F)

(J) = (F)

$x + 3y \quad\quad = -7$ (K)

$-x + y \quad\quad = -1$ (L)

$x + 2y + 3z = 13$ (M)

(K) = (G)

(L) = (H) $\div$ (-4)

(M) = (J)

$4y \quad\quad = -8$ (N)

$-x + y \quad\quad = -1$ (P)

$x + 2y + 3z = 13$ (Q)

(N) = (K) + (L)

(P) = (L)

(Q) = (M)

LOOK FOR RELATIONSHIPS
The following actions do not change the solutions of a system of equations:

- rearranging the order of the list

- multiplying an equation by a nonzero number

- adding one equation to another and replacing one of these equations with the sum

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From equation (N):

$y = -2$

Substituting into (P):

$-x + (-2) = -1$

$x = -1$

Substituting into (A):

$2(-1) + (-2) - z = -10$

$z = 6$

The solution is $(-1, -2, 6)$.

B. What is the solution of this system of equations? $\begin{cases} 2x - y + z = 3 \\ x + y + z = 5 \\ -4x + 2y - 2z = 0 \end{cases}$

$2x - y + z = 3$ (A)

$x + y + z = 5$ (B)

$-4x + 2y - 2z = 0$ (C)

$3x + \quad 2z = 8$ (D)

$x + y + z = 5$ (E)

$-6x \quad\quad -4z = -10$ (F)

(D) = (A) + (B)

(E) = (B)

(F) = -2(B) + (C)

$3x + \quad 2z = 8$ (G)

$x + y + z = 5$ (H)

$0 = 6$ (J)

(G) = (D)

(H) = (E)

(J) = 2(D) + (F)

Equation (J) is not true. There is no solution for this system of equations; it is an **inconsistent system**.

CONTINUED ON THE NEXT PAGE

EXAMPLE 3 CONTINUED

Concept Summary | Assess

✓ **Try It!**   **3.** Solve the following systems of equations.

a. $\begin{cases} x + y + z = 3 \\ x - y + z = 1 \\ x + y - z = 2 \end{cases}$

b. $\begin{cases} 2x + y - 2z = 3 \\ x - 2y + 7z = 12 \\ 3x - y + 5z = 10 \end{cases}$

🔑 CONCEPT SUMMARY   Linear Systems

	System of linear equations	System of linear inequalities
WORDS	a set of two or more equations using the same variables	a set of two or more inequalities using the same variables
ALGEBRA	$\begin{cases} 4x - 3y = 4 \\ -x + 2y = 5 \end{cases}$	$\begin{cases} y \geq 16 - 2x \\ y \leq 16 - x \\ y \geq 6 \end{cases}$
GRAPHS		

✓ Do You UNDERSTAND?

1. 🔲 **ESSENTIAL QUESTION**   How can you find and represent solutions of systems of linear equations and inequalities?

2. **Error Analysis** Shandra said the solution of the system of equations $\begin{cases} 2x + y = 3 \\ -x + 4y = -6 \end{cases}$ is $(-1, 2)$. Is she correct? Explain.

3. **Communicate Precisely** Why is a system of linear inequalities often solved graphically?

4. **Make Sense and Persevere** How does knowing how to solve a system of two equations in two variables help you to solve a system of three equations in three variables?

5. **Vocabulary** What is the difference between a system of linear equations and a system of linear inequalities?

Do You KNOW HOW?

6. Solve the following system of equations.
$\begin{cases} 2x + 2y = 10 \\ x + 5y = 13 \end{cases}$

7. Graph the following system of inequalities.
$\begin{cases} -x + 2y < 1 \\ x \geq 0 \\ y \geq 0 \end{cases}$

8. Solve the following system of equations.
$2x - y + z = 3$
$3x + y + 3z = 10$
$x - 2y - 2z = 3$

9. Equations with two variables that are raised only to the first power represent lines. There are three possible outcomes for the intersections of two lines. Describe the outcomes.

UNDERSTAND

10. Communicate Precisely Consider a point that lies on the border of the shaded region of the graph of a system of linear inequalities. Under what conditions is that point a solution to the system?

11. Error Analysis Describe and correct the error a student made in solving the system of equations.

$$2x + 4y = 0 \implies 2x + 4y = 0$$
$$3x - 2y = -24 \implies 6x - 4y = -24$$
$$\Downarrow$$
$$8x = -24$$
$$x = -3$$
$$\Downarrow$$
$$2(-3) + 4y = 0$$
$$-6 + 4y = 0$$
$$4y = 6$$
$$y = \frac{3}{2} \quad ✗$$

12. Higher Order Thinking Suppose that solving a system of equations algebraically gives the following result: $0 = 0$

What does this mean graphically?

13. Use Structure Write a system of equations in three variables with integer solutions. Give the solution. Explain your process.

14. Make Sense and Persevere Write a system of inequalities for the shaded region.
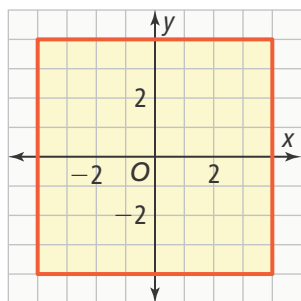
15. Mathematical Connections Consider the following system of equations.

$$\begin{cases} x = 5 - 3y \\ y = -2x \end{cases}$$

Write a system of inequalities whose solution includes the solution to the system of equations above.

PRACTICE

Solve the following systems of equations.
SEE EXAMPLE 1

16. $\begin{cases} x = 2y - 5 \\ 3x - y = 5 \end{cases}$

17. $\begin{cases} y = 2x + 3 \\ 2y - x = 12 \end{cases}$

18. $\begin{cases} x - 3y = 1 \\ 2x - y = 7 \end{cases}$

19. $\begin{cases} x + 2y = -4 \\ 3x - y = -5 \end{cases}$

Write a system of linear equations that has the solution shown.

20. $(-3, 5)$

21. $(10, -1)$

Sketch the graph of the set of all points that solve each system of linear inequalities. SEE EXAMPLE 2

22. $\begin{cases} 0 < x \le 125 \\ x \ge 2y > 0 \\ 2x + 2y \le 300 \end{cases}$

23. $\begin{cases} y + 2x < 10 \\ x - 2y < 8 \\ x > 0 \\ y > 0 \end{cases}$

24. $\begin{cases} y \le -2x + 19 \\ y \ge \frac{3}{7}x + 2 \\ x \le 7 \end{cases}$

25. $\begin{cases} y < \frac{3}{2}x \\ 3x + 2y < 36 \\ 3 < y < 6 \end{cases}$

26. Charles has a collection of dimes and quarters worth $1.25. He has 8 coins. Write a system of equations to represent this situation. Then solve the system to determine how many dimes and how many quarters Charles has. SEE EXAMPLE 2

27. A set of triangular and square tiles contains 50 pieces and 170 sides. Write a system of equations to represent this situation. Then solve the system to determine how many triangular and how many square tiles there are. SEE EXAMPLE 2

Solve the following systems of equations.
SEE EXAMPLE 3

28. $\begin{cases} 2x - y - 3z = 20 \\ 3x + y + 6z = 4 \\ x + 2y + 9z = -16 \end{cases}$
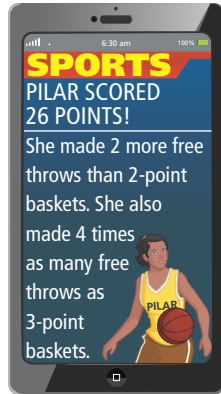
29. $\begin{cases} 2x + 5y - 3z = 14 \\ x - 2y + 4z = -12 \\ -x + 3y - 2z = 13 \end{cases}$

**30.** $\begin{cases} 8x - y + 2z = 1 \\ -2x + 3y + 7z = 8 \\ 4x - 2y + z = -4 \end{cases}$

31. $\begin{cases} 7x - y - 5z = -27 \\ 2x + y + 3z = -19 \\ x + 3y + z = 5 \end{cases}$

PRACTICE & PROBLEM SOLVING

APPLY

32. Model With Mathematics In basketball, a successful free throw is worth 1 point, a basket made from inside the 3-point arc is worth 2 points, and a basket made from outside the 3-point arc basket is worth 3 points. How many of each type of basket did Pilar make?

SPORTS
PILAR SCORED 26 POINTS!
She made 2 more free throws than 2-point baskets. She also made 4 times as many free throws as 3-point baskets.

33. Reason Raul is paid $75 per week plus $5 for each new gym membership he sells. He may switch to a gym that pays $50 per week and $7.50 for each new membership. How many memberships per week does Raul have to sell for the new gym to be a better deal for him?

34. Reason Keisha is designing a rectangular giraffe enclosure with a length of at most 125 m. The animal sanctuary can afford at most 300 m of fencing, and the length of the enclosure must be at least double the width.

$y \leq 125$ m
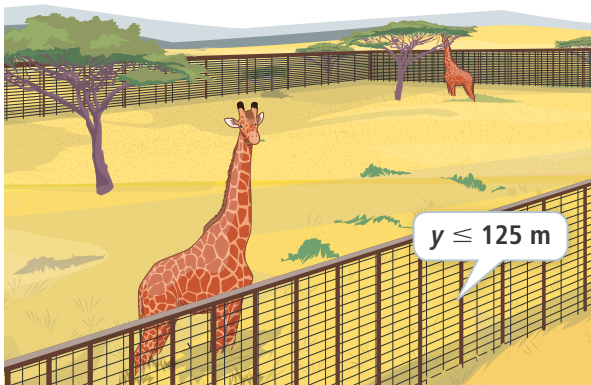
a. Write inequalities to represent each constraint where x = width and y = length.

b. Graph and solve the linear system of inequalities.

c. What does the solution mean?

35. Make Sense and Persevere Ramona needs 10 mL of a 30% saline solution. She has a 50% saline solution and a 25% saline solution. How many milliliters of each solution does she need to create the 30% solution?

ASSESSMENT PRACTICE

36. One equation in a system of equations with one solution is $4x + 2y = 14$. Select all equations that could be the second equation in the system. **A-REI.3.6**

☐ **A.** $2x + y = 7$
☐ **B.** $3x - 6y = -12$
☐ **C.** $2x + 6y = 32$
☐ **D.** $-3x + 10y = 1$
☐ **E.** $2x + y = 5$

37. SAT/ACT What value of a gives $(-1, 1)$ as the solution of the system $\begin{cases} 3x + 5y = 2 \\ ax + 8y = 14 \end{cases}$?

Ⓐ -22 Ⓑ -6 Ⓒ 0 Ⓓ 6 Ⓔ 22

38. Performance Task Each Sophomore, Junior, and Senior at a high school collected aluminum cans and plastic bottles. The table shows the average number of cans and bottles collected per student, by grade level during a 3 week recyling drive.

Week 1 2,319 cans and bottles
Week 3 3,785 cans and bottles
Week 2 2,290 cans and bottles

	Sophomores	Juniors	Seniors
Week 1	3	4	4
Week 2	4	4	3
Week 3	5	6	7

Part A Write a system of equations to represent the situation.

Part B Find the solution of the system of equations you found in Part A.

Part C What does your solution to part B represent in terms of this scenario?