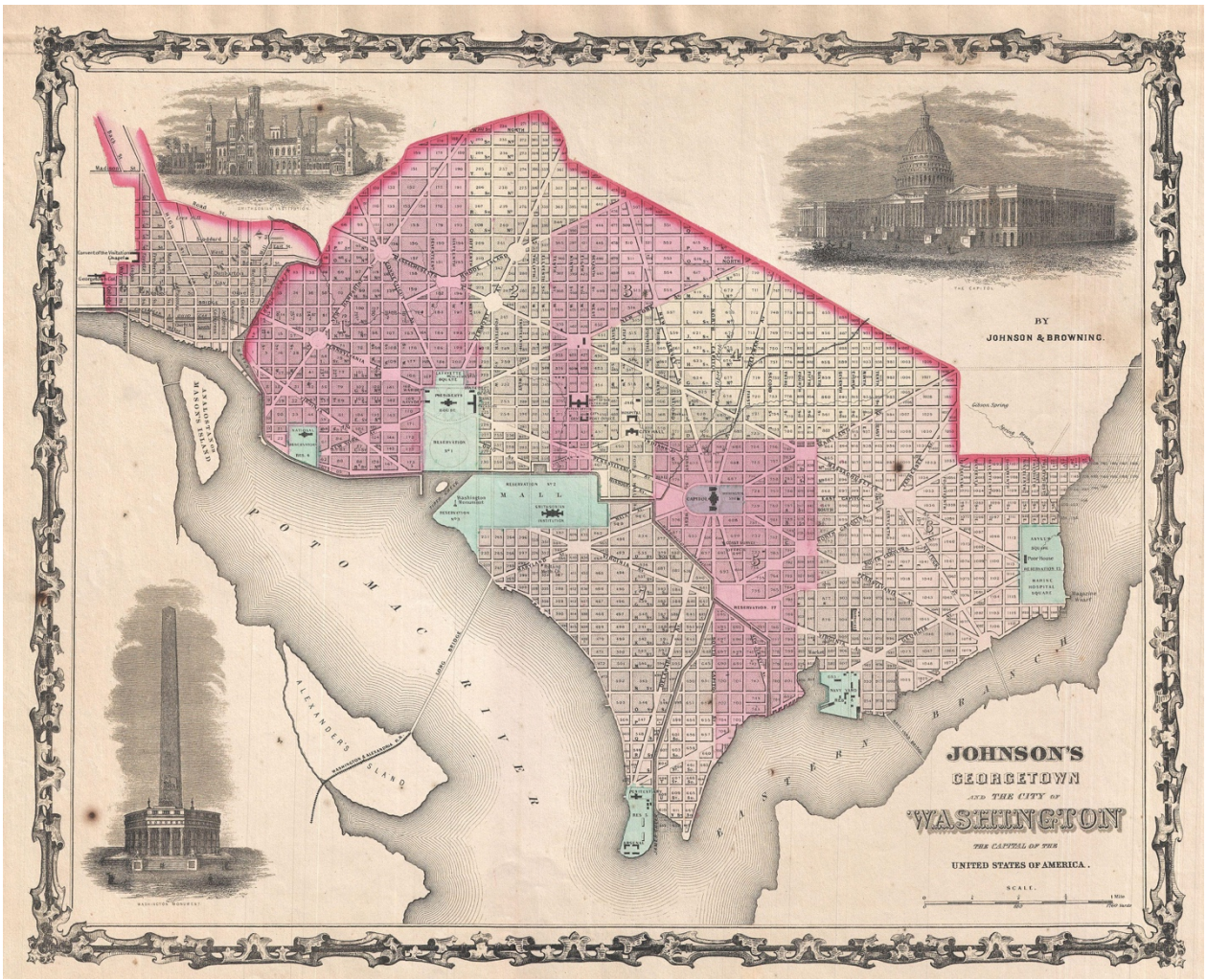
Passport to Advanced Mathematics



Solving Quadratic Equations

Basic Example

What are all the solutions to the equation below?

$$4x^2 = 52$$

(A) $x = 13$

(B) $x = -13$ and $x = 13$

(C) $x = \sqrt{13}$

(D) $x = -\sqrt{13}$ and $x = \sqrt{13}$

Harder Example

What are the solutions to the equation below?

$$(x + 3)(x - 5) = 5$$

(A) $x = -3$ and $x = 5$

(B) $x = 2$ and $x = 10$

(C) $x = 1 - 2\sqrt{5}$ and $x = 1 + 2\sqrt{5}$

(D) $x = 1 - \sqrt{21}$ and $x = 1 + \sqrt{21}$

Interpreting Nonlinear Expressions

Basic Example

Kleiber's law, below, relates the basal metabolic rate, B, measured in kilocalories per day, of an animal to its body mass, M, measured in kilograms. If the body mass of an elephant is 10^6 times that of a mouse, which of the following best compares their basal metabolic rates?

$$B = 70M^{0.75}$$

(A) The basal metabolic rate of the elephant is 0.75 times that of the mouse

(B) The basal metabolic rate of the mouse is $10^{4.5}$ times that of the elephant

(C) The basal metabolic rate of the elephant is $10^{4.5}$ times that of the mouse

(D) The basal metabolic rate of the elephant is 10^6times that of the mouse

Harder Example

The following equation shoes Kepler's 3rd law of planetary motion. It relates the time, *t* (in days), that a planet takes to revolve once around our sun to the distance, *d* (in kilometers), of that planet from the sun.

$$t^2 = 3.98 \cdot 10^{-20} \cdot d^3$$

Mars is approximately 4 times as distant from the sun as Mercury is. About how many times longer would Mars's revolution time be than for Mercury? Round your answer to the nearest whole number.

Quadratic and Exponential Word Problems

Basic Example

A cable company with a reputation for poor customer service is losing subscribers at a rate of approximately 3% per year. The company had 2 million subscribers at the start of 2014.

Assume that the company continues to lose subscribers at the same rate, and that there are no new subscribers. Which of the following functions, S, models the number of subscribers (in millions) remaining t years after the start of 2014?

(A) $S(t) = 2(1.03)^t$

(B) $S(t) = 2(0.97)^t$

(C) $S(t) = 2(0.70)^t$

(D) $S(t) = 2(0.97)t$

Harder Example

Currently, a local newspaper company sells print subscriptions for \$9.30 a month and has 2400 subscribers. Based on a survey conducted, they expect to lose 20 subscribers for each \$0.10 increase from the current monthly subscription price. What should the newspaper company charge for a monthly subscriptions price. What should the newspaper compay charge for a monthly subscription in order to maximize the income from the print newspaper subscriptions?

(A) \$1.35

(B) \$9.30

(C) \$10.65

(D) \$22.80

Manipulating Quadratic and Exponential Expressions

Basic Example

Zac plans to buy a new car, but she has learned that her car will lose value quickly in the first few years that she owns it. An online calculator uses the following formula to show her value, $V(t)$, in dollars, of the car t years from now.

$$V(t) = 24{,}900 \cdot 0.85^t$$

How many times its present value would Zac's car be worth after any 2 years period?

(A) 0.7225

(B) 0.85

(C) 0.9219

(D) 1.7

Harder Example

Dina purchased 200 feet of fencing to make a rectangular play area for her dogs. The possible area, A, is given by the equation below where w is the width of the play area.

$$A(w) = 100w - w^2$$

Which of the following equivalent expressions displays, as a constant or coefficient, the value of the width for which the area is a maximum?

(A) $-(w - 50)^2 + 2500$

(B) $-(w + 20)^2 + 140w + 400$

(C) $-(w - 10)^2 + 80w + 100$

(D) $-(w + 10)^2 + 120w + 100$

Radicals and Rational Exponents

Basic Example

Which of the following expressions is equivalent to

$$\sqrt[3]{27x^4y^6}$$

(A) $3xy^2$

(B) $3x^{\frac{4}{3}}y^2$

(C) $3x^2y^3$

(D) $9xy^2$

Harder Example

Which of the following values is equal to the value below?

$$3^{-\frac{1}{5}} \cdot \left(\frac{1}{3}\right)^{-\frac{2}{5}}$$

(A) $3^{-\frac{2}{25}}$

(B) $3^{-\frac{1}{5}}$

(C) 1

(D) $3^{\frac{1}{5}}$

Radicals and Rational Equations

Basic Example

What is the solution to the equation below?

$$2 = \frac{11}{4k - 3}$$

(A) $k = -\frac{17}{8}$

(B) $k = -\frac{8}{17}$

(C) $k = \frac{8}{17}$

(D) $k = \frac{17}{8}$

Harder Example

What is the sum of all the solutions to the above equation?

$$3 + \sqrt{6m - 26} = m$$

Operations with Rational Expressions

Basic Example

Which expression is equivalent to the following quotient for all $a > 0, b > 0, \text{and } c > 0$

$$\frac{\left(\frac{3a^2b^5}{c^3}\right)}{\left(\frac{15ab^2}{25c^4}\right)}$$

(A) $\frac{9a^4b^5}{5c^7}$

(B) $\frac{9a^3b^{10}}{5c^{12}}$

(C) $\frac{a^2b^3}{5c}$

(D) $5ab^3c$

Note: Sal messed up in his example on Khan (which is noted in the website) The correct answer is given above.

Harder Example

Which expression is equivalent to the difference shown below?

$$\frac{3}{x^2+5x-24} - \frac{7}{x-3}$$

(A) $\frac{-4}{x^2+4x-27}$

(B) $\frac{-4}{x^2+4x-21}$

(C) $\frac{-7x-53}{x^2+5x-24}$

(D) $\frac{-7x+59}{x^2+5x-24}$

Operations with Polynomials

Basic Example

Which of the following expressions is equivalent to $7k$ minus the product of $k+1$ and $2k+2$

(A) $-2k^2+3k-2$

(B) $5k^2+3k-2$

(C) $6k^2+10k+2$

(D) $12k^2+10k+2$

Harder Example

Which of the following is equivalent to $\left(\frac{1}{88}y^{100}+1\right) - \left(\frac{1}{44}y^{100}-\frac{1}{2}\right)$?

(A) $-\frac{1}{88}y^{100}+\frac{1}{2}$

(B) $-\frac{1}{88}y^{100}+\frac{3}{2}$

(C) $\frac{1}{44}y^{100}+\frac{1}{2}$

(D) $\frac{1}{44}y^{100}+\frac{3}{2}$

Polynomial Factors and Graphs

Basic Example

Given the polynomial below, what are its zeros?

$$P(x) = (x + 7)(x - 10)$$

(A) $x = -70, x = -3$, and $x = 1$

(B) $x = -1, x = 3$, and $x = 70$

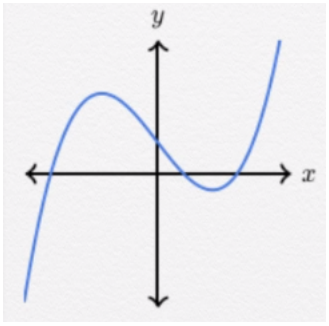
(C) $x = -7$ and $x = 10$

(D) $x = -10$ and $x = 7$

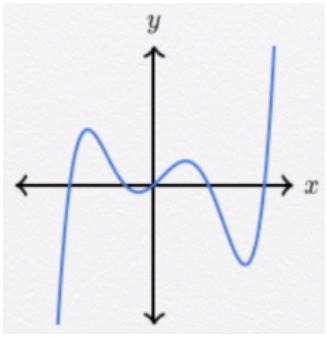
Harder Example

The polynomial $p(x)$ has 4 distinct zeros. Which of the following graphs could represent $y = p(x)$?

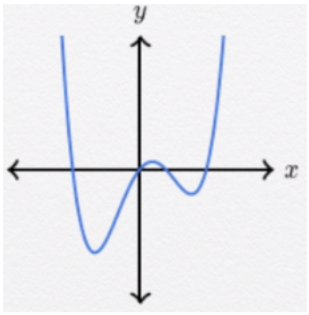
(A)



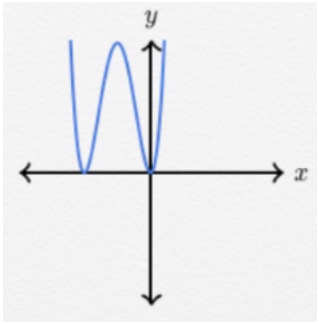
(B)



(C)



(D)



Nonlinear Equation Graphs

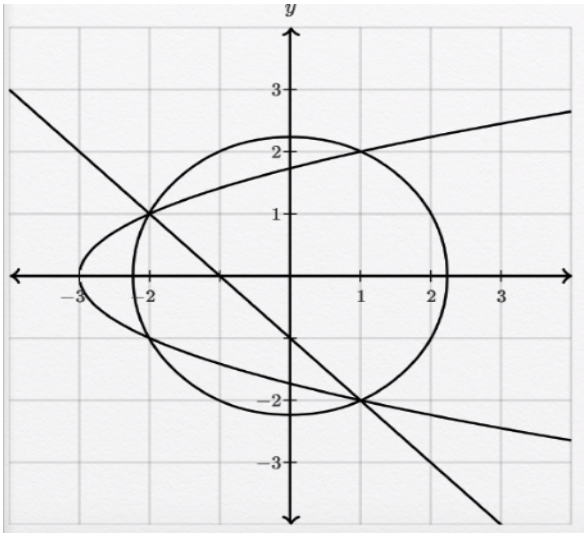
Basic Example

A system of equations and their graphs in the xy-plane are shown below. How many solutions does the system have?

$$y^2 = x + 3$$

$$y + x = -1$$

$$x^2 + y^2 = 5$$



(A) One

(B) Two

(C) Three

(D) Four

Harder Example

In the xy – plane, which of the following is a solution to the system of inequalities below?

$$y > \left[\frac{1}{2}\right]^x$$

$$y < -\frac{1}{2}x^2 + 2$$

(A) (0,2)

(B) (1,1)

(C) (2,1)

(D) (3,0)

Linear and Quadratic Systems

Basic Example

Which of the following represents all solutions (x, y) to the system of equations shown below

$$2y + 6 = x$$
$$y^2 - 9 = x$$

(A) (-5,3)

(B) (5,-3)

(C) (16, 5) and (0,-3)

(D) (5, 16) and (-3, 0)

Harder Example

If (x, y) is a solution to the system of equations shown below, what is the product of the x- coordinates of the solutions?

$$x^2 + x + 5 = y$$
$$2x + 7 = y$$

Structure in Expressions

Basic Example

Which of the following is equivalent to the expression below?

$$(p + 1)^2 - 4$$

(A) $(p + 1 + 2)(p + 1 - 2)$

(B) $(p - 1 + 2)(p - 1 - 2)$

(C) $(p + 1 + 4)(p + 1 - 4)$

(D) $(p - 1 + 4)(p - 1 - 4)$

Harder Example

Which of the following is equivalent to the expression below?

$$a^2\left(\frac{1}{2x - y}\right)^2 - 1$$

(A) $\frac{a^2}{4x^2 - y^2} - 1$

(B) $\left(\frac{a}{2x - y} - 1\right)^2$

(C) $\left(\frac{a}{2x - y} - 1\right)\left(\frac{1}{2x - y} + 1\right)$

(D) $\left(\frac{a}{2x - y} - 1\right)\left(\frac{a}{2x - y} + 1\right)$

Isolating Quantities

Basic Example

The absolute pressure, *P,* in a fluid of density, p, at a given depth, *h,* can be found with the equation below, where P_o is atmospheric pressure and *g* is gravitational acceleration. Which of the following is the correct expression for the depth in terms of the absolute pressure, atmospheric pressure, fluid density, and gravitational acceleration?

$$P = P_o + pgh$$

(A) $h = \frac{P - P_o}{pg}$

(B) $h = \frac{P + P_o}{pg}$

(C) $h = \frac{P}{pg} - P_o$

(D) $h = \frac{P}{pg} + P_o$

Harder Example

If an initial investment, P, bears interest at a rate, r, and is compounded annually, its future value, *A,* after *t* years can be determined with the equation below. Which of the following equations shows the interest rate in terms of the future value, initial investment, and number of years invested.

$$A = P(1 + r)^t$$

(A) $r = \frac{\frac{A}{P} - 1}{t}$

(B) $r = \frac{A - P - 1}{t}$

(C) $r = \left(\frac{A}{P} - 1\right)^{\frac{1}{t}}$

(D) $r = \left(\frac{A}{P}\right)^{\frac{1}{t}} - 1$

Function Notation

Basic Example

Let $g(x) = x^2 - 5$. If $f\big(g(x)\big) = \sqrt{x^2 + 4}$, which of the following describes $f(x)$?

(A) $f(x) = \sqrt{x + 1}$

(B) $f(x) = \sqrt{x + 9}$

(C) $f(x) = \sqrt{x^2 + 1}$

(D) $f(x) = \sqrt{x^2 + 9}$

Harder Example

Let $y(x) = \sqrt[3]{x^3 + 1}$. Which of the following is equivalent to $y\big(y(x)\big)$?

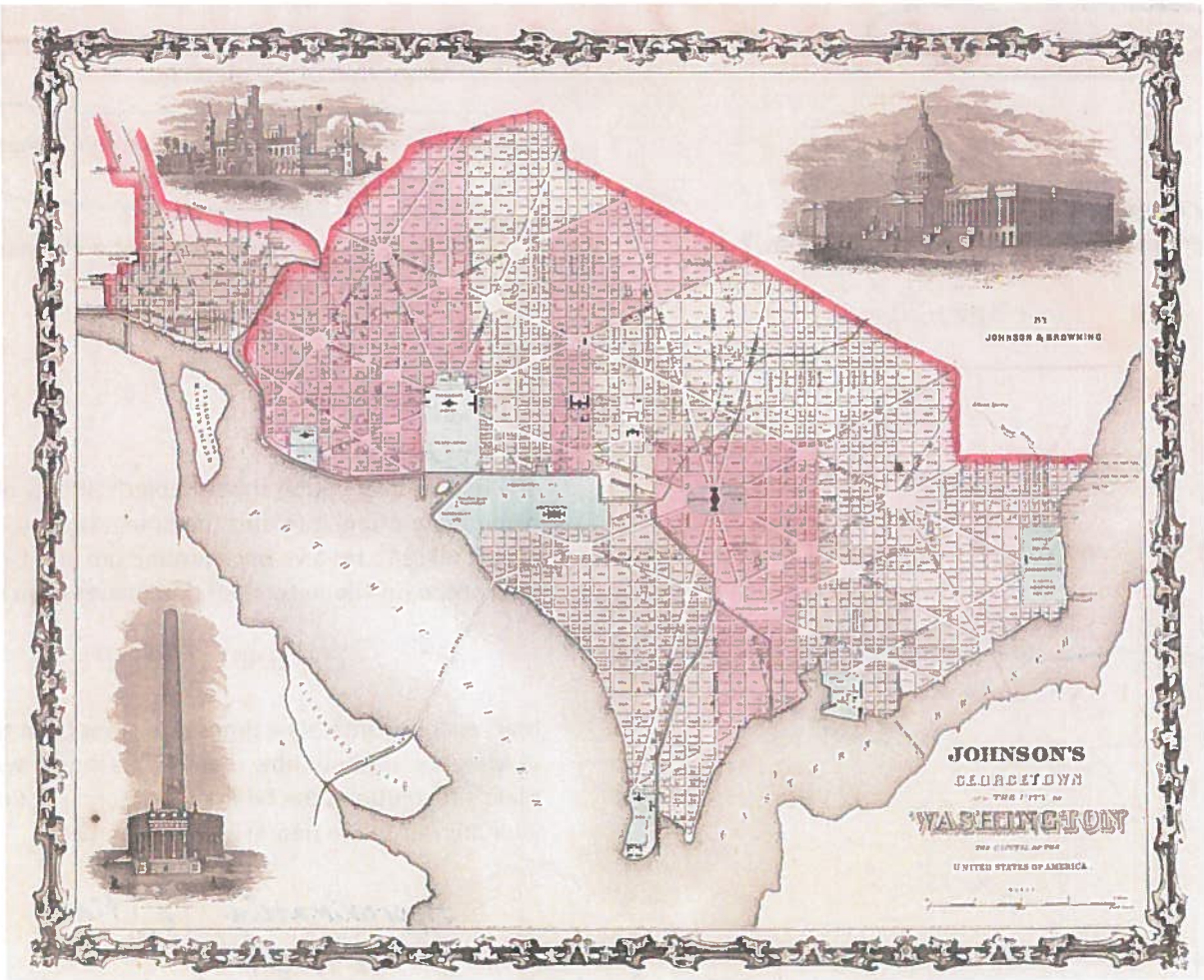
(A) $\sqrt[3]{x^3 + 1}$

(B) $\sqrt[3]{x^3 + 1} + 1$

(C) $\sqrt[3]{x^3 + 2}$

(D) $\sqrt[3]{(x^3 + 1)^3 + 1}$

Passport to Advanced Mathematics



Solving Quadratic Equations

Basic Example

What are all the solutions to the equation below?

$$4x^2 = 52$$

$4x^2 = 52$

$x^2 = 13$

$x = \pm\sqrt{13}$

(A) $x = 13$

(B) $x = -13$ and $x = 13$

$4(-\sqrt{13})^2 = 52$ ✓

$4(\sqrt{13})^2 = 52$ ✓

(C) $x = \sqrt{13}$

(D) $x = -\sqrt{13}$ and $x = \sqrt{13}$

Harder Example

What are the solutions to the equation below?

$$(x + 3)(x - 5) = 5$$

(A) $x = -3$ and $x = 5$

(B) $x = 2$ and $x = 10$

(C) $x = 1 - 2\sqrt{5}$ and $x = 1 + 2\sqrt{5}$

(D) $x = 1 - \sqrt{21}$ and $x = 1 + \sqrt{21}$

FOIL $(x+3)(x-5) = 5$

$x^2 - 5x + 3x - 15 = 5$

$x^2 - 2x - 20 = 0$

$a^2 + bx + c = 0$

$a = 1$, $b = -2$, $c = -20$

Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-20)}}{2(1)}$

$= \frac{2 \pm \sqrt{4 + 80}}{2}$

$= \frac{2}{2} \pm \frac{\sqrt{84}}{2} \rightarrow = 1 \pm \frac{\sqrt{4}\sqrt{21}}{2} = \boxed{1 \pm \sqrt{21}}$

Interpreting Nonlinear Expressions

Basic Example

Kleiber's law, below, relates the basal metabolic rate, B, measured in kilocalories per day, of an animal to its body mass, M, measured in kilograms. If the body mass of an elephant is 10^6 times that of a mouse, which of the following best compares their basal metabolic rates?

$$B = 70M^{0.75}$$

$= 70(10^6)^{.75} = 70(10^{4.5})$

(A) The basal metabolic rate of the elephant is 0.75 times that of the mouse

(B) The basal metabolic rate of the mouse is $10^{4.5}$ times that of the elephant

(C) The basal metabolic rate of the elephant is $10^{4.5}$ times that of the mouse

(D) The basal metabolic rate of the elephant is 10^6 times that of the mouse

Harder Example

The following equation shoes Kepler's 3rd law of planetary motion. It relates the time, t (in days), that a planet takes to revolve once around our sun to the distance, d (in kilometers), of that planet from the sun.

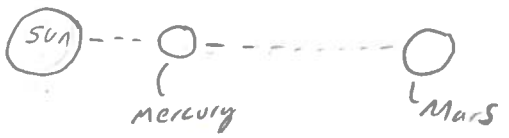
$$t^2 = 3.98 \cdot 10^{-20} \cdot d^3$$

Mars is approximately 4 times as distant from the sun as Mercury is. About how many times longer would Mars's revolution time be than for Mercury? Round your answer to the nearest whole number.

Approximately [8] times longer

t = time (days)

d = distance of planet to sun



$t^2 = 3.98 \cdot 10^{-20} \cdot 4^3$

$t = \sqrt{3.98 \cdot 10^{-20} \cdot 64}$

$t = \sqrt{3.98 \cdot 10^{-20}} \cdot 8$

$\therefore$ 8 times longer

Quadratic and Exponential Word Problems

Basic Example

A cable company with a reputation for poor customer service is losing subscribers at a rate of approximately 3% per year. The company had 2 million subscribers at the start of 2014.

Assume that the company continues to lose subscribers at the same rate, and that there are no new subscribers. Which of the following functions, S, models the number of subscribers (in millions) remaining t years after the start of 2014?

$S = 2(.97)^t$

$= 2(.97)^2$

$= .9409$

(A) $S(t) = 2(1.03)^t$

(B) $S(t) = 2(0.97)^t$

(C) $S(t) = 2(0.70)^t$

(D) $S(t) = 2(0.97)t$

$S(0) = 2(.97)^0 = 2$

$S(1) = 2(.97)^1 = 2(.97)$

$S(2) = 2(.97)^2 = 2(.97)(.97)$

Harder Example

Currently, a local newspaper company sells print subscriptions for \$9.30 a month and has 2400 subscribers. Based on a survey conducted, they expect to lose 20 subscribers for each \$0.10 increase from the current monthly subscription price. What should the newspaper company charge for a monthly subscriptions price. What should the newspaper compay charge for a monthly subscription in order to maximize the income from the print newspaper subscriptions?

$I = S \cdot P$

I = Income

S = Subscribers

P = price

(A) \$1.35

(B) \$9.30

(C) \$10.65

(D) \$22.80

$S = 2400 - 20\left(\frac{P - 9.30}{.10}\right)$

$S = 2400 - 200(P - 9.30)$

$S = 2400 - 200P + 1860$

$S = 4260 - 200P$

$I = S \cdot P$

$I = (4260 - 200P)P$

$I = -200P^2 + 4260P$

Vertex of Parabolla $= \frac{-b}{2a} = \frac{-4260}{2(-200)} = 10\frac{13}{20}$ $\quad a = -200 \quad b = 4260$

Manipulating Quadratic and Exponential Expressions

Basic Example

Zac plans to buy a new car, but she has learned that her car will lose value quickly in the first few years that she owns it. An online calculator uses the following formula to show her value, $V(t)$, in dollars, of the car t years from now.

$$V(t) = 24{,}900 \cdot 0.85^t$$

How many times its present value would Zac's car be worth after any 2 years period?

$.85^2 = .7225$

(A) 0.7225

(B) 0.85

(C) 0.9219

(D) 1.7

Harder Example

Dina purchased 200 feet of fencing to make a rectangular play area for her dogs. The possible area, A, is given by the equation below where w is the width of the play area.

$$A(w) = 100w - w^2$$

Which of the following equivalent expressions displays, as a constant or coefficient, the value of the width for which the area is a maximum?

$100w - w^2 = 0$

$w(100 - w) = 0$

$\therefore w = 0$ & $w = 100$

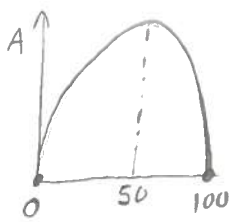
$\therefore$ we need 50 b/c it's the vertex

(A) $-(w - 50)^2 + 2500$

(B) $-(w + 20)^2 + 140w + 400$

(C) $-(w - 10)^2 + 80w + 100$

(D) $-(w + 10)^2 + 120w + 100$



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Basic Example

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$$\sqrt[3]{27x^4y^6}$$

(A) $3xy^2$

(B) $3x^{\frac{4}{3}}y^2$

(C) $3x^2y^3$

(D) $9xy^2$

$= \sqrt[3]{27} \cdot \sqrt[3]{x^4} \cdot \sqrt[3]{y^6}$

$= 3^{3\cdot\frac{1}{3}} \cdot x^{4\cdot\frac{1}{3}} \cdot y^{6\cdot\frac{1}{3}}$

$= 3^1 \cdot x^{\frac{4}{3}} \cdot y^{\frac{6}{3}}$

$= \boxed{3x^{\frac{4}{3}}y^2}$

Harder Example

Which of the following values is equal to the value below?

$$3^{-\frac{1}{5}} \cdot \left(\frac{1}{3}\right)^{-\frac{2}{5}}$$

(A) $3^{-\frac{2}{25}}$

(B) $3^{-\frac{1}{5}}$

(C) 1

(D) $3^{\frac{1}{5}}$

$= 3^{-\frac{1}{5}} \cdot (3^{-1})^{-\frac{2}{5}}$

$= 3^{-\frac{1}{5}} \cdot 3^{\frac{2}{5}}$

$= 3^{-\frac{1}{5}+\frac{2}{5}}$

$= 3^{\frac{1}{5}}$

Radicals and Rational Equations

Basic Example

What is the solution to the equation below?

$$4k-3\left[2 = \frac{11}{4k-3}\right]4k-3$$

(A) $k = -\frac{17}{8}$

(B) $k = -\frac{8}{17}$

(C) $k = \frac{8}{17}$

(D) $k = \frac{17}{8}$

$(4k-3)2 = 11$

$8k-6 = 11$

$8k = 17$

$\boxed{k = \frac{17}{8}}$

Harder Example

What is the sum of all the solutions to the above equation?

$$3 + \sqrt{6m-26} = m$$

$m=7$ & $m=5$

Therefore, $7+5 = \boxed{12}$

$3 + \sqrt{6m-26} = m$

$\sqrt{6m-26} = m-3$

$6m-26 = (m-3)^2$ $\quad (m-3)(m-3)$ m^2-6m+9

$6m-26 = m^2-6m+9$

$-6m \qquad -6m$

$-26 = m^2-12m+9$

$0 = m^2-12+35$

$(m-7)(m-5)$

$\therefore m=7$ & $m=5$

$\therefore$ sum is $7+5=\boxed{12}$

Operations with Rational Expressions

Basic Example

Which expression is equivalent to the following quotient for all $a > 0, b > 0,$ and $c > 0$

$$\frac{\left(\frac{3a^2b^5}{c^3}\right)}{\left(\frac{15ab^2}{25c^4}\right)}$$

(A) $\frac{9a^4b^5}{5c^7}$

(B) $\frac{9a^3b^{10}}{5c^{12}}$

(C) $\frac{a^2b^3}{5c}$

(D) $5ab^3c$

$= \frac{3a^2b^5}{c^3} \times \frac{25c^4}{15ab^2}$

$= \frac{75a^2b^5c^4}{15ab^2c^3}$

$= 5ab^3c$

Note: Sal messed up in his example on Khan (which is noted in the website) The correct answer is given above.

Harder Example

Which expression is equivalent to the difference shown below?

$$\frac{3}{x^2+5x-24} - \frac{7}{x-3}$$

(A) $\frac{-4}{x^2+4x-27}$

(B) $\frac{-4}{x^2+4x-21}$

(C) $\frac{-7x-53}{x^2+5x-24}$

(D) $\frac{-7x+59}{x^2+5x-24}$

$= \frac{3}{(x+8)(x-3)} - \frac{7}{x-3}$

$= \frac{3}{(x+8)(x-3)} - \frac{7(x+8)}{(x+8)(x-3)}$

$= \frac{3-7(x+8)}{(x+8)(x-3)}$

$= \frac{3-7x-56}{(x+8)(x-3)}$

$= \frac{-7x-53}{(x+8)(x-3)}$

Operations with Polynomials

Basic Example

Which of the following expressions is equivalent to $7k$ minus the product of $k+1$ and $2k+2$

(A) $-2k^2+3k-2$

(B) $5k^2+3k-2$

(C) $6k^2+10k+2$

(D) $12k^2+10k+2$

$7k - (k+1)(2k+2)$

$7k - [2k^2 + 4k + 2]$

$7k - 2k^2 - 4k - 2$

$-2k^2 + 3k - 2$

Harder Example

Which of the following is equivalent to $\left(\frac{1}{88}y^{100}+1\right) - \left(\frac{1}{44}y^{100} - \frac{1}{2}\right)$?

(A) $-\frac{1}{88}y^{100}+\frac{1}{2}$

(B) $-\frac{1}{88}y^{100}+\frac{3}{2}$

(C) $\frac{1}{44}y^{100}+\frac{1}{2}$

(D) $\frac{1}{44}y^{100}+\frac{3}{2}$

$\frac{1}{88}y^{100} + 1 - \frac{1}{44}y^{100} + \frac{1}{2}$

$\left(\frac{1}{88} - \frac{1}{44}\right)y^{100} + \frac{3}{2}$

$\left(\frac{1}{88} - \frac{2}{88}\right)y^{100} + \frac{3}{2}$

$-\frac{1}{88}y^{100} + \frac{3}{2}$

Polynomial Factors and Graphs

Basic Example

Given the polynomial below, what are its zeros?

$$P(x) = (x + 7)(x - 10)$$

$x+7=0$ $\quad x-10=0$

$x=-7$ $\quad x=10$

(A) $x = -70, x = -3$, and $x = 1$

(B) $x = -1, x = 3$, and $x = 70$

(C) $x = -7$ and $x = 10$

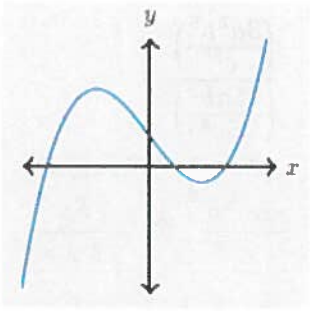
(D) $x = -10$ and $x = 7$

Harder Example

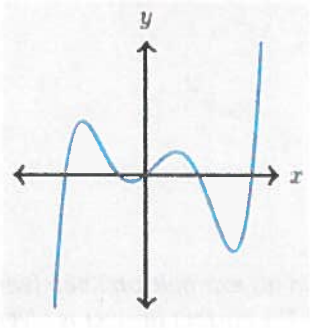
The polynomial $p(x)$ has 4 distinct zeros. Which of the following graphs could represent $y = p(x)$?

B/c the function has 4 distinct zeros, it must cross the x-axis 4 times

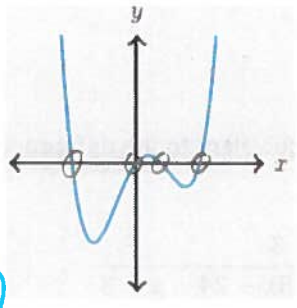
(A)



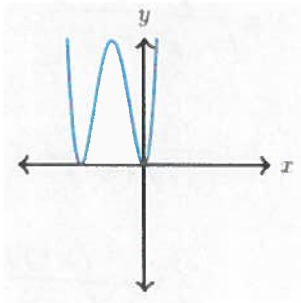
(B)



(C)



(D)



Nonlinear Equations Graphs

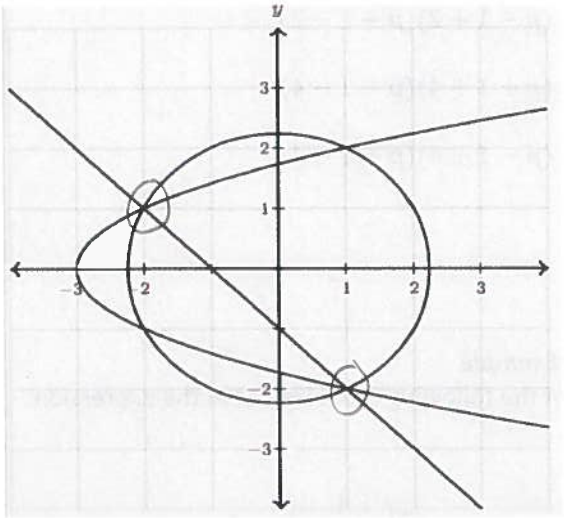
Basic Example

A system of equations and their graphs in the xy-plane are shown below. How many solutions does the system have?

$$y^2 = x + 3$$

$$y + x = -1$$

$$x^2 + y^2 = 5$$



(A) One

(B) Two

(C) Three

(D) Four

Must hit all 3 functions. This happens in 2 places

Harder Example

In the xy – plane, which of the following is a solution to the system of inequalities below?

$$y > \left[\frac{1}{2}\right]^x$$

$$y < -\frac{1}{2}x^2 + 2$$

(A) (0,2)

(B) (1,1)

(C) (2,1)

(D) (3,0)

Test each pt.

X (0,2) $2 > \left(\frac{1}{2}\right)^0$? $2 < -\frac{1}{2}0^2 + 2$?

x y $2 > 1$ ✓ $2 < 2$ X

✓ (1,1) $1 > \left(\frac{1}{2}\right)^1$? $1 < -\frac{1}{2}(1)^2 + 2$

$1 > \frac{1}{2}$ ✓ $1 < -\frac{1}{2} + 2$

$1 < 1\frac{1}{2}$ ✓

The point (1,1) satisfies both constraints

Nonlinear Equations Graphs

Basic Example

Which of the following represents all solutions (x, y) to the system of equations shown below

$$2y + 6 = x$$
$$y^2 - 9 = x$$

$2y+6 = y^2-9$

$6 = y^2-2y-9$

$0 = y^2-2y-15$

$0 = (y-5)(y+3)$

$y=5$ $y=-3$

y_1 y_2

(A) (-5,3)

(B) (5,-3)

(C) (16, 5) and (0,-3)

(D) (5, 16) and (-3, 0)

Harder Example

If (x, y) is a solution to the system of equations shown below, what is the product of the x- coordinates of the solutions?

$$x^2 + x + 5 = y$$
$$2x + 7 = y$$

$x=2$ & $x=-1$

Product $= 2 \cdot -1 = \boxed{-2}$

$x^2+x+5 = 2x+7$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$ & $x=-1$

Product: $2 \times -1 = \boxed{-2}$

Structure in Expressions

Basic Example

Which of the following is equivalent to the expression below?

$$(p + 1)^2 - 4$$

$a^2+b^2 = (a+b)(a-b)$

$(p+1)^2 - 2^2 = (p+1+2)(p+1-a)$

a b

(A) $(p + 1 + 2)(p + 1 - 2)$

(B) $(p - 1 + 2)(p - 1 - 2)$

(C) $(p + 1 + 4)(p + 1 - 4)$

(D) $(p - 1 + 4)(p - 1 - 4)$

Harder Example

Which of the following is equivalent to the expression below?

$$a^2\left(\frac{1}{2x - y}\right)^2 - 1$$

$a^2-b^2 = (a+b)(a-b)$

$= \left(a\frac{1}{2x-y}\right)^2 - 1^2$

a b

$= \left(\frac{a}{2x-y}\right)^2 - 1^2$

$= \left(\frac{a}{2x-y}+1\right)\left(\frac{a}{2x-y}-1\right)$

(A) $\frac{a^2}{4x^2-y^2} - 1$

(B) $\left(\frac{a}{2x-y} - 1\right)^2$

(C) $\left(\frac{a}{2x-y} - 1\right)\left(\frac{1}{2x-y} + 1\right)$

(D) $\left(\frac{a}{2x-y} - 1\right)\left(\frac{a}{2x-y} + 1\right)$

Isolating Quantities

Basic Example

The absolute pressure, P, in a fluid of density, p, at a given depth, h, can be found with the equation below, where P_o is atmospheric pressure and g is gravitational acceleration. Which of the following is the correct expression for the depth in terms of the absolute pressure, atmospheric pressure, fluid density, and gravitational acceleration?

$$P = P_o + pgh$$

(A) $h = \frac{P-P_o}{pg}$

(B) $h = \frac{P+P_o}{pg}$

(C) $h = \frac{P}{pg} - P_o$

(D) $h = \frac{P}{pg} + P_o$

$P - P_o = pgh$

$\frac{P-P_o}{pg} = h$

$h = \frac{P-P_o}{pg}$

Harder Example

If an initial investment, P, bears interest at a rate, r, and is compounded annually, its future value, A, after t years can be determined with the equation below. Which of the following equations shows the interest rate in terms of the future value, initial investment, and number of years invested.

$$A = P(1+r)^t$$

(A) $r = \frac{\frac{A}{P}-1}{t}$

(B) $r = \frac{A-P-1}{t}$

(C) $r = \left(\frac{A}{P}-1\right)^{\frac{1}{t}}$

(D) $r = \left(\frac{A}{P}\right)^{\frac{1}{t}} - 1$

$\frac{A}{P} = (1+r)^t$

$(1+r)^t = \frac{A}{P}$

$(1+r)^{t \cdot \frac{1}{t}} = \left(\frac{A}{P}\right)^{\frac{1}{t}}$

$1+r = \left(\frac{A}{P}\right)^{1/t}$

$r = \left(\frac{A}{P}\right)^{1/t} - 1$

Function Notation

Basic Example

Let $g(x) = x^2 - 5$. If $f(g(x)) = \sqrt{x^2+4}$, which of the following describes $f(x)$?

(A) $f(x) = \sqrt{x+1}$

(B) $f(x) = \sqrt{x+9}$

(C) $f(x) = \sqrt{x^2+1}$

(D) $f(x) = \sqrt{x^2+9}$

$f(g(x)) = \sqrt{x^2+4}$

$f(g(x)) = \sqrt{g(x)+1}$

$= \sqrt{x^2-5+1}$ ✗

$f(g(x)) = \sqrt{g(x)+9}$

$= \sqrt{x^2-5+9}$

$= \sqrt{x^2+4}$ ✓

Harder Example

Let $y(x) = \sqrt[3]{x^3+1}$. Which of the following is equivalent to $y(y(x))$?

$\left(\sqrt[3]{a}\right)^3 = a$

$y(y(x)) = \sqrt[3]{(y(x))^3+1}$

$= \sqrt[3]{\sqrt[3]{(x^3+1)}^3+1}$

$= \sqrt[3]{x^3+1+1}$

$= \sqrt[3]{x^3+2}$

(A) $\sqrt[3]{x^3+1}$

(B) $\sqrt[3]{x^3+1}+1$

(C) $\sqrt[3]{x^3+2}$

(D) $\sqrt[3]{(x^3+1)^3+1}$