Name: **Date:**

Topic: 5.1-Nth Roots, Radicals, and Rational Exponents (Class Notes)

Essential Question: How are exponents and radicals used to represent roots of real numbers?

Questions/Key Ideas	Notes/Problems/Work

Powers & Roots

2nd power: 2^2 (square)

3rd power: 2^3 (cube)

4th power: 2^4

Square root: $\sqrt{2}$ (index = 2, usually not written)

Cube root: $\sqrt[3]{2}$ (index = 3)

4th root: $\sqrt[4]{2}$ (index = 4)

$\sqrt{2}$ ← radical symbol

$\sqrt[3]{2}$ ← radicand (the inside)

$\sqrt[4]{80} = \sqrt[4]{16}\cdot\sqrt[4]{5} = \boxed{2\sqrt[4]{5}}$

$3\sqrt[5]{64} = 3\sqrt[5]{32}\cdot\sqrt[5]{2} = 3\cdot 2\sqrt[5]{2} = \boxed{6\sqrt[5]{2}}$

Rational Exponents

$\sqrt[3]{2^1} = 2^{1/3}$

$\sqrt[5]{2^4} = 2^{4/5}$

$\sqrt[\text{index}]{2^{\text{power}}} = 2^{\frac{\text{power}}{\text{index}}}$

$2^{-3/4}$ — Flip & Forget (sign)

$\frac{2^{-3}}{1} = \frac{1}{2^{3/4}} = \frac{1}{\sqrt[4]{2^3}}$

Example 1 Convert between radical and exponential form. Simplify. Find the value on the calculator if possible.

(a) $2^{\frac{5}{2}}$ $\sqrt[2]{2^5} = \sqrt[2]{2^4}\cdot\sqrt[2]{2^1} = 2\sqrt{2^2}\cdot\sqrt{2} = \boxed{2\sqrt[2]{4}}$

(b) $6^{\frac{3}{2}}$ $\sqrt{6^3} = \sqrt{6^2}\cdot\sqrt{6^1} = \boxed{6\sqrt{6}}$

(c) $(\sqrt[4]{5})^5$ $\sqrt[4]{5^5}$ $5^{5/4}$ $5^{1\frac{1}{4}}$ $= 5^1\cdot 5^{1/4}$ $\boxed{5\sqrt[4]{5}}$

(d) $\sqrt[3]{2}$ $2^{1/3} \approx 1.26$

$2^{0.5} = 2^{1/2} = \sqrt{2}$

Questions/Key Ideas	Notes/Problems/Work

Example 2

Convert between radical and exponential forms. Do not simplify.

(a) $(\sqrt[4]{m})^3$ $m^{3/4}$

(b) $\frac{1}{(\sqrt{2x})^3}$ $\frac{1}{(2x)^{3/2}} = (2x)^{-3/2}$

(c) $(10n)^{\frac{3}{2}}$

$\sqrt{(10n)^3}$

or $(\sqrt{10n})^3$

(d) $(5x)^{-\frac{3}{4}}$ $\frac{1}{(5x)^{3/4}} = \frac{1}{\sqrt[4]{(5x)^3}}$

Simplify Radicals

$$\sqrt[n]{x^n} = \begin{cases} |x|, & n \text{ is even} \\ x, & n \text{ is odd} \end{cases}$$

$\sqrt[4]{2^4} = 2$ $\sqrt[4]{x^4} = |x|$

$\sqrt[3]{2^3} = 2$ $\sqrt[3]{x^3} = x$

$\sqrt[4]{(-2)^4} = \sqrt[4]{16} = \sqrt[4]{2^4} = 2$

$\sqrt[5]{(-2)^5} = \sqrt[5]{-32} = -2$

Example 3

Simplify the following radical expressions. Use absolute values where appropriate.

(a) $\sqrt[4]{48a^2b^4}$ 4 even

$\sqrt{48}\sqrt{a^2}\sqrt{b^4}$

$\sqrt{2^4}\sqrt{2^4}\sqrt{3}\sqrt{a^2}\sqrt{b^4}$

$= 2 \cdot 2\sqrt{3}|a||b|^2$

$= 4\sqrt{3}|ab^2|$

(b) $\sqrt[5]{32x^5y^6z}$

(c) $2\sqrt[3]{54y^4z^8}$ OR $= 4\sqrt{3}|a|b^2$

(d) $5\sqrt[4]{80x^5y^8z^{13}} = 5\sqrt{80}\sqrt{x^4}\sqrt{x}\sqrt{y^8}\sqrt{(z^{12})^2}$

$5\sqrt{2^4}\sqrt{5}\sqrt{x^4}\sqrt{x}\sqrt{y^8}\sqrt{(z^{12})^2}$

$5 \cdot 2 \cdot 2|x|\sqrt{x}\sqrt{y^8}|z^{12}|$

$20|x|z^{12}|\sqrt{xy}$

Summary (Address EQ):