Activity Assess

1-3 Piecewise-Defined Functions

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I CAN… graph and interpret piecewise-defined functions.

VOCABULARY
- piecewise-defined function
- step function

MAFS.912.F-IF.3.7.b–Graph... piecewise-defined functions, including step functions and absolute value functions.
Also F-IF.2.5
MAFS.K12.MP.3.1, MP.6.1, MP.7.1

MODEL & DISCUSS

A music teacher needs to buy guitar strings for her class. At store A, the guitar strings cost $6 each. At store B, the guitar strings are $20 for a pack of 4.

A. Make graphs that show the income each store receives if the teacher needs 1–20 guitar strings

B. Describe the shape of the graph for store A. Describe the shape of the graph for store B. Why are the graphs different?

C. **Communicate Precisely** Compare the graphs for stores A and B. For what numbers of guitar strings is it cheaper to buy from store B? Explain how you know.

ESSENTIAL QUESTION How do you model a situation in which a function behaves differently over different parts of its domain?

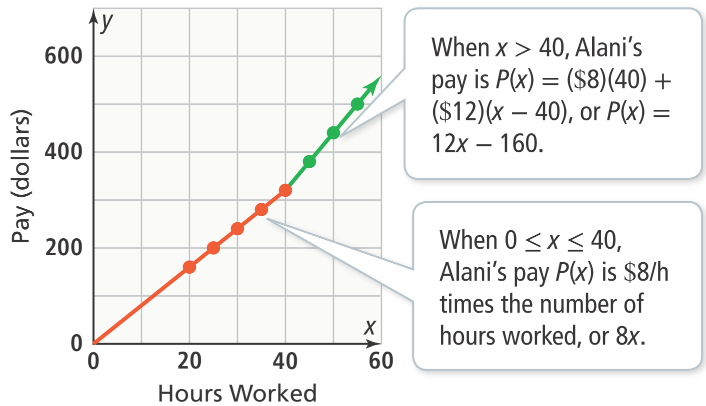
CONCEPTUAL UNDERSTANDING

EXAMPLE 1 Model With a Piecewise-Defined Function

Alani has a summer job as a lifeguard. She makes $8/h for up to 40 h each week. If she works more than 40 h, she makes 1.5 times her hourly pay, or $12/h, for each hour over 40 h. How could you make a graph and write a function that shows Alani's weekly earnings based on the number of hours she worked?

Step 1 Make a table of values and a graph.

Hours Worked	Pay
20	160
25	200
30	240
35	280
40	320
45	380
50	440
55	500



STUDY TIP
Remember that Alani makes $8/h for the first 40 h and $12/h for any additional hours after that.

Step 2 Notice that the plot contains two linear segments with a slope that changes slightly at $x = 40$. A function that has different rules for different parts of its domain is called a **piecewise-defined function**.

Step 3 Write an equation for each piece of the graph.

$$P(x) = \begin{cases} 8x, & 0 \le x \le 40 \\ 12x - 160, & x > 40 \end{cases}$$

USE STRUCTURE
This notation is used for piecewise-defined functions to indicate the different functions at different parts of the domain.

CONTINUED ON THE NEXT PAGE

EXAMPLE 1 CONTINUED

Try It! **1.** How much will Alani earn if she works:

a. 37 hours? **b.** 43 hours?

EXAMPLE 2 Graph a Piecewise-Defined Function

How do you graph a piecewise defined function?

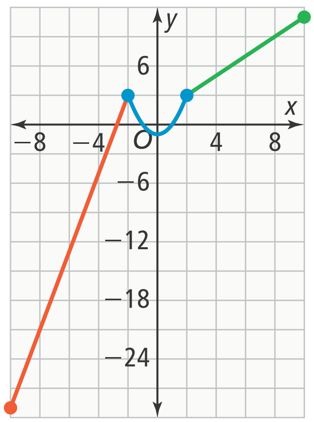
$$f(x) = \begin{cases} 4x + 11, & -10 \le x < -2 \\ x^2 - 1, & -2 \le x \le 2 \\ x + 1, & 2 < x \le 10 \end{cases}$$

What are the domain and range? Over what intervals is the function increasing or decreasing?

Sketch the graph of $y = 4x + 11$ for values of x between −10 and −2.

Sketch the graph of $y = x^2 - 1$ for values of x between −2 and 2.

Sketch the graph of $y = x + 1$ for values between 2 and 10.



To determine the range, calculate the y-values that correspond to the minimum and maximum x-values on the graph. For this graph, these values occur at the endpoints of the domain of the piecewise function, $-10 \le x \le 10$.

Evaluate $y = 4x + 11$ for $x = -10$

$y = 4(-10) + 11$

$y = -29$

Evaluate $y = x + 1$ for $x = 10$

$y = 10 + 1$

$y = 11$

The range is $-29 \le y \le 11$.

The domain is $\{x | -10 \le x \le 10\}$. The range is $\{y \mid -29 \le y \le 11\}$. The function is increasing when $-10 < x < -2$ and $0 < x < 10$. The function is decreasing when $-2 < x < 0$.

COMMON ERROR
The values of −2 and 2 are only included in one piece of the graph. If they were included in more than one piece and had different values for different pieces, this would not be a function.

Try It! **2.** Graph the piecewise-defined function. What are the domain and range? Over what intervals is the function increasing or decreasing?

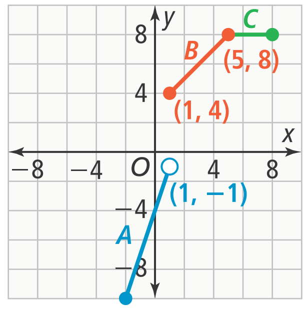
a. $f(x) = \begin{cases} 2x + 5, & -6 \le x \le -2 \\ 2x^2 - 7, & -2 < x < 1 \\ -4 - x, & 1 \le x \le 3 \end{cases}$

b. $f(x) = \begin{cases} 3, & -4 < x \le 0 \\ -x, & 0 \le x \le 2 \\ 3 - x, & 2 < x < 4 \end{cases}$

EXAMPLE 3 Write a Piecewise-Defined Rule From a Graph

STUDY TIP
A closed circle on the graph means the coordinates of the point are included in the domain and range of the function. An open circle indicates they are not included.

What is the rule that describes the piecewise-defined function shown in the graph?



Step 1 Notice three separate linear pieces that make up the function.

Step 2 Determine the domain of each segment.

Step 3 For each segment, use the graph to locate points on the line and to find the slope.

Step 4 You can use the slope-intercept form of a linear function, $f(x) = mx + b$, to define the function for each segment.

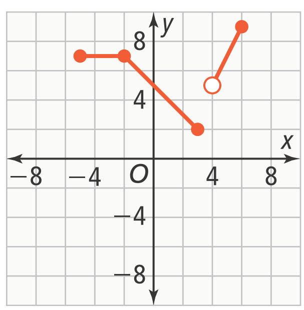
Segment A	Segment B	Segment C
Domain: $-2 \le x < 1$	**Domain: $1 \le x \le 5$**	**Domain: $5 < x \le 8$**
(1, −1), slope = 3	(1, 4), slope = 1	(5, 8), slope = 0
$y = mx + b$ $-1 = (3)(1) + b$ $b = -4$	$y = mx + b$ $4 = (1)(1) + b$ $b = 3$	$y = mx + b$ $8 = (0)(5) + b$ $b = 8$
$f(x) = 3x - 4$	$f(x) = x + 3$	$f(x) = 8$

The rule for this function is:

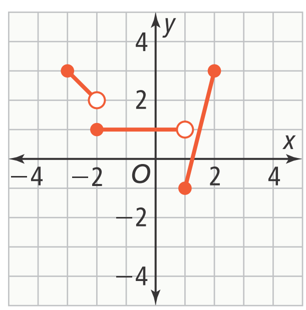
$$f(x) = \begin{cases} 3x - 4, & -2 \le x < 1 \\ x + 3, & 1 \le x \le 5 \\ 8, & 5 < x \le 8 \end{cases}$$

Try It! **3.** What rule defines the function in each of the following graphs?

a.



b.



EXAMPLE 4 Write a Rule for an Absolute Value Function

How can you rewrite the function $f(x) = |6x + 18|$ as a piecewise-defined fuction?

Step 1 Write the function in the form $f(x) = a|x - h| + k$ to find the vertex of the function.

$$f(x) = |6x + 18|$$
$$= |6(x + 3)|$$
$$= 6|x - (-3)| + 0$$

$k = 0$

$h = -3$

The vertex is $(h, k) = (-3, 0)$. The graph has two linear pieces, one to the left of $x = -3$, and one to the right of $x = -3$.

Step 2 Determine the slope and equation of each piece of the function by testing x-values on either side of -3.

	Choose a point so that $x < -3$: let $x = -4$	Choose a point so that $x > -3$: let $x = 0$
Point	$(-4, 6)$	$(0, 18)$
Slope to (−3, 0)	-6	6
Equation	$y = -6x - 18$	$y = 6x + 18$

GENERALIZE
The parent absolute value function $f(x) = |x|$ is a piecewise-defined function:

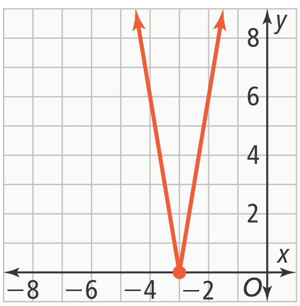
$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Step 3 Write the piecewise-defined function.

The absolute value function $f(x) = |6x + 18|$ can be written as the piecewise-defined function:

$$f(x) = \begin{cases} -6x - 18, & x < -3 \\ 6x + 18, & x \geq -3 \end{cases}$$

Step 4 Confirm by graphing.



The vertex is (−3, 0).

Try It! **4.** How can you rewrite each function as a piecewise-defined function?

a. $f(x) = |-5x - 10|$ **b.** $f(x) = -|x| + 3$

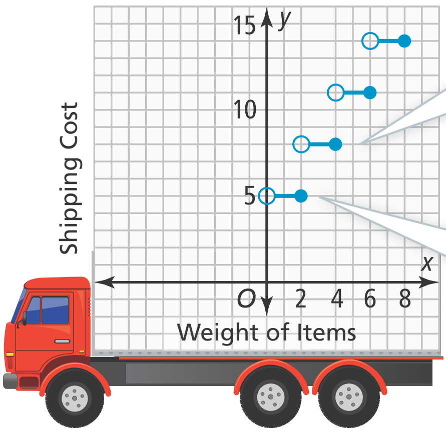
APPLICATION

EXAMPLE 5 Graph a Step Function

The shipping cost of items purchased from an online store is dependent on the weight of the items. The table represents shipping costs y based on the weight x. Graph the function. What are the domain and range of the function? What are the maximum and minimum values?

Weight of Items	$0 < x \leq 2$ lb	$2 < x \leq 4$ lb	$4 < x \leq 6$ lb	$6 < x \leq 8$ lb
Shipping Cost	\$5	\$8	\$11	\$14

The graph of the function looks like the steps of a staircase. This is called a **step function** since it pairs every input in an interval with the same output value.



Each y-value goes with a set interval of x-values. For example, $y = 5$ for $0 < x \leq 2$ and $y = 8$ for $2 < x \leq 4$.

A closed circle means that the x-value is included in that domain, and the $\leq$ or $\geq$ sign is used. An open circle means it is not included.

COMMON ERROR
You might think that the range of this function would be the interval [5, 14], but only the values 5, 8, 11, and 14 are possible outputs.

Domain: $\{x \mid 0 < x \leq 8\}$

Range: $\{5, 8, 11, 14\}$

This function has a minimum of 5 and a maximum of 14.

Try It! **5.** The table below represents fees for a parking lot. Graph the function. What are the domain and range of the function? What are the maximum and minimum values?

Time	$0 < t \leq 3$h	$3 < t \leq 6$h	$6 < t \leq 9$h	$9 < t \leq 12$h
Cost	\$10	\$15	\$20	\$25

CONCEPT SUMMARY Piecewise-Defined Functions

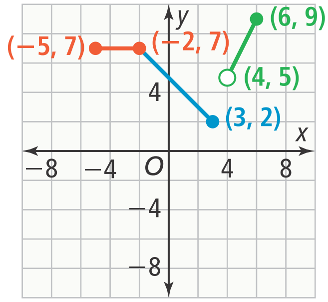
WORDS

A piecewise function has different rules for different parts of its domain.

ALGEBRA

$$f(x) = \begin{cases} 7, & -5 \le x \le -2 \\ 5 - x, & -2 < x \le 3 \\ 2x - 3, & 4 < x \le 6 \end{cases}$$

GRAPH



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do you model a situation in which a function behaves differently over different parts of its domain?

2. **Vocabulary** How do piecewise-defined functions differ from step functions?

3. **Error Analysis** Given the function
$$f(x) = \begin{cases} 2x + 5, & -2 < x \le 4 \\ -4x - 7, & 4 < x \le 9 \end{cases}$$
Rebecca says there is an open circle at $x = 4$ for both pieces of the function. Explain her error.

4. **Communicate Precisely** What steps do you follow when graphing a piecewise-defined function?

5. **Make Sense and Persevere** Is the relation defined by the following piecewise rule a function? Explain.
$$y = \begin{cases} 7x - 4, & x < 2 \\ -x + 5, & x \ge -2 \end{cases}$$

Do You KNOW HOW?

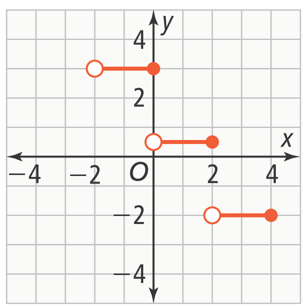
Graph the function.

6. $f(x) = \begin{cases} -x + 1, & -10 \le x < -3 \\ x^2 - 9, & -3 \le x \le 3 \\ 2x + 1, & 3 < x < 5 \end{cases}$

7. $g(x) = \begin{cases} 1, & 0 \le x < 2 \\ 3, & 2 \le x < 4 \\ 5, & 4 \le x < 6 \\ 7, & 6 \le x < 8 \end{cases}$

8. Given the function
$$f(x) = \begin{cases} -2x + 4, & 0 \le x < 8 \\ -5x + 11, & x \ge 8 \end{cases}$$
is the function increasing or decreasing over the interval [2, 7]? Find the rate of change over this interval.

9. What is the rule that defines the function shown in the graph?

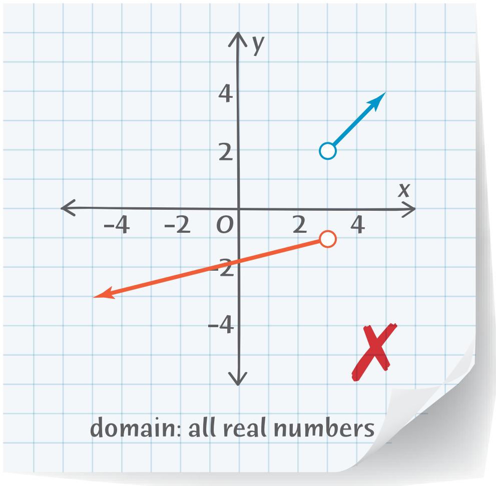


PRACTICE & PROBLEM SOLVING

UNDERSTAND

10. **Communicate Precisely** What do closed circles and open circles on the graph of a step function indicate?

11. **Error Analysis** What error did Damian make when defining the domain of the graph? Explain.



12. **Communicate Precisely** For what values of x is the function $f(x) = \begin{cases} -3x + 4, & -2 < x \leq 3 \\ 2x + 1, & 4 \leq x < 9 \end{cases}$ defined?

13. **Mathematical Connections** For the piecewise-defined function $f(x) = \begin{cases} 7, & x > 3 \\ 5x - 3, & x \leq 3 \end{cases}$ find two x-values that have the same y-value and the sum of the x-values is 10.

14. **Higher Order Thinking** The function $f(x) = \lfloor x \rfloor$ is called the greatest integer function because the output returned is the greatest integer less than or equal to x. For example, $f(3.2) = \lfloor 3.2 \rfloor = 3$ and $f(0.975) = \lfloor 0.975 \rfloor = 0$. Graph the function $f(x) = \lfloor x \rfloor$. What type of graph does this look like?

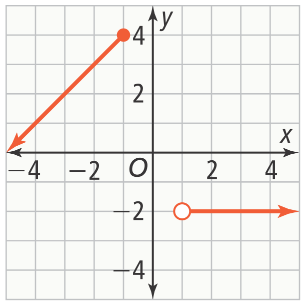
PRACTICE

15. A phone company offers a monthly cellular phone plan for \$25. The plan includes 250 anytime minutes, and charges \$0.20 per minute above 250 min. Write a piecewise-defined function for $C(x)$, the cost for using x minutes in a month. SEE EXAMPLE 1

16. Graph the piecewise-defined function. State the domain and range. Identify whether the function is increasing, constant, or decreasing on each interval of the domain. SEE EXAMPLE 2

$$f(x) = \begin{cases} \frac{1}{4}x + 3, & -2 < x \leq 0 \\ 2, & 0 < x \leq 4 \\ 3 - x, & 4 < x \leq 7 \end{cases}$$

17. Write the rule that defines the function in the following graph. SEE EXAMPLE 3



Write each absolute value function as a piecewise-defined function. SEE EXAMPLE 4

18. $f(x) = |3x + 1|$

19. $g(x) = |-2x - 6|$

Graph the step function. SEE EXAMPLE 5

20. $f(x) = \begin{cases} 2, & -3 \leq x < 1 \\ 5, & 1 \leq x < 4 \\ 8, & 4 \leq x < 6 \\ 9, & 6 \leq x < 10 \end{cases}$

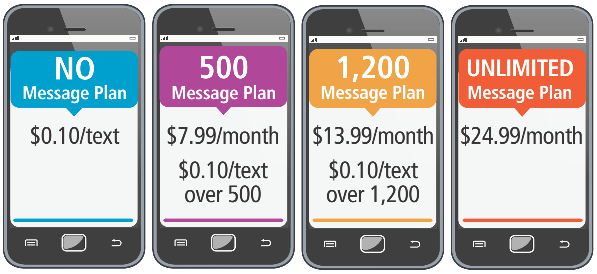
21. The parking rates for a parking garage are shown. Graph the function for the cost of parking rates at the garage. SEE EXAMPLE 5

PRACTICE & PROBLEM SOLVING

APPLY

22. **Model With Mathematics** If Kyle works more than 40 h per week, his hourly wage for the extra hour(s) is 1.5 times the normal hourly wage of \$10 per hour. Write a piecewise-defined function that gives Kyle's weekly pay P in terms of the number h of hours he works. Determine how much Kyle will get paid if he works 45 h.

23. **Model With Mathematics** Text message plans offered at a phone company, along with overage charges, are shown.



a. Write a function for each plan where x is the number of texts and $f(x)$ is the total monthly cost.

b. Sarah uses approximately 1,500 texts per month. What is the monthly cost under each text message plan?

c. Write an interval for the number of text messages that would make each plan the best one to purchase.

24. **Reason** The cost C (in dollars) of sending next-day mail depends on the weight x (in ounces) of a package. The cost of packages, up to 5 lb, is given by the function below. What are the domain and range of the function?

$$f(x) = \begin{cases} 12.25, & 0 < x \leq 8 \\ 16.75, & 8 < x \leq 32 \\ 19.50, & 32 < x \leq 48 \\ 23.50, & 48 < x \leq 64 \\ 25.25, & 64 < x \leq 80 \end{cases}$$

ASSESSMENT PRACTICE

25. Use the functions below to construct a piecewise-defined function f on the intervals $(-\infty, -1)$, $[-1, 2]$, and $(2, \infty)$ that has a graph that is completely connected. Then draw the graph. F-IF.3.7.b

$$f(x) = \begin{cases} \underline{\quad ? \quad}, & \text{if } x < -1 \\ \underline{\quad ? \quad}, & \text{if } -1 \leq x \leq 2 \\ \underline{\quad ? \quad}, & \text{if } x > 2 \end{cases}$$

$g_1(x) = x$ $g_2(x) = -x$ $g_3(x) = x^2$

$g_4(x) = -x^2$ $g_5(x) = 4$ $g_6(x) = 2$

26. **SAT/ACT** What is the vertex of the absolute value function $f(x) = -|x - a| + b$ where a and b are real numbers?

Ⓐ (a, b)
Ⓑ $(-a, b)$
Ⓒ $(a, -b)$
Ⓓ $(-a, -b)$

27. **Performance Task** Yama works a varying number of hours per month for a construction company. The following scatter plot shows how much money he earns for each number of hours he works. Write the piecewise-defined function that represents Yama's earnings as a function of his hours worked.