2-2 Standard Form of a Quadratic Function

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I CAN… write and graph quadratic functions in standard form.

VOCABULARY

- standard form of a quadratic function

MAFS.912.F-IF.2.4–For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Also A-CED.1.2
MAFS.K12.MP.3.1, MP.4.1, MP.7.1

CRITIQUE & EXPLAIN

Jordan and Emery are rewriting the vertex form of the quadratic function $y = 2(x - 4)^2 + 5$ in the form $y = ax^2 + bx + c$.

Jordan	Emery
$y = 2(x - 4)^2 + 5$	$y = 2(x - 4)^2 + 5$
$= (2x - 8)^2 + 5$	$= 2(x^2 - 16) + 5$
$= 4x^2 - 32x + 64 + 5$	$= 2x^2 - 32 + 5$
$= 4x^2 - 32x + 69$	$= 2x^2 - 27$

A. **Communicate Precisely** Did Jordan rewrite the equation correctly? Did Emery? Explain.

B. Without rewriting the equation, how could you prove that Jordan or Emery's equations are not equivalent to the original?

ESSENTIAL QUESTION

What key features can you determine about a quadratic function from an equation in standard form?

EXAMPLE 1 Find the Vertex of a Quadratic Function in Standard Form

CONCEPTUAL UNDERSTANDING

How can you find the vertex of a quadratic function written in standard form?

A. What is the x-coordinate of the vertex of $f(x) = ax^2 + bx + c$?

The **standard form of a quadratic function** is $y = ax^2 + bx + c$ where a, b, and c are real numbers, and $a \neq 0$. Use vertex form to derive standard form.

$y = a(x - h)^2 + k$ ········ Write the vertex form of a quadratic equation.

$y = a(x^2 - 2xh + h^2) + k$ ········ Square the binomial.

$y = ax^2 - 2ahx + ah^2 + k$ ········ Simplify.

The equation $y = ax^2 - 2ahx + ah^2 + k$ is a *quadratic function* in standard form with $a = a$, $b = -2ah$, and $c = ah^2 + k$.

The vertex of a quadratic function is (h, k), so to determine the x-coordinate of the vertex, solve $b = -2ah$ for h.

$$b = -2ah$$

$$-\frac{b}{2a} = h$$

Since h is the x-coordinate of the vertex, you can use this value to find the y-value, k, of the vertex.

LOOK FOR RELATIONSHIPS
By converting vertex form into standard form, you can see how h and k relate to the coefficients of the equation.

B. What is the vertex of the function $f(x) = x^2 - 6x + 10$?

Step 1 Identify the coefficients a, b, and c.

$a = 1$, $b = -6$, and $c = 10$

Step 2 Solve for h, the x-coordinate of the vertex.

$$h = -\frac{b}{2a} = -\frac{(-6)}{2(1)} = 3$$

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EXAMPLE 1 CONTINUED

Step 3 Substitute the value of h into the equation for x to find k, the y-coordinate of the vertex.

$$\begin{aligned} f(3) &= (3)^2 - 6(3) + 10 \\ &= 9 - 18 + 10 \\ &= 1 \end{aligned}$$

The vertex of the function is $(h, k) = (3, 1)$.

Try It! **1.** What is the vertex of the graph of the function $f(x) = x^2 - 8x + 5$?

EXAMPLE 2 Graph a Quadratic Function in Standard Form

How can you use key features to graph $f(x) = x^2 - 4x + 8$?

For $f(x)$, identify a, b, and c: $a = 1$, $b = -4$, and $c = 8$.

Step 1 Find the vertex and the axis of symmetry of the quadratic function.

The x-coordinate of the vertex and the axis of symmetry can be determined by:

$$h = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2$$

Substitute the value of h for x into the equation to find the y-coordinate of the vertex, k:

$$f(2) = (2)^2 - 4(2) + 8 = 4$$

The vertex is $(2, 4)$, and the axis of symmetry is $x = 2$.

USE STRUCTURE
The y-intercept of a quadratic function in standard form is given by the ordered pair $(0, c)$. Verify this by substituting $x = 0$ into the standard form equation.

Step 2 Find the y-intercept of the quadratic function.

The y-intercept occurs at

$$f(0) = (0)^2 - 4(0) + 8 = 8.$$

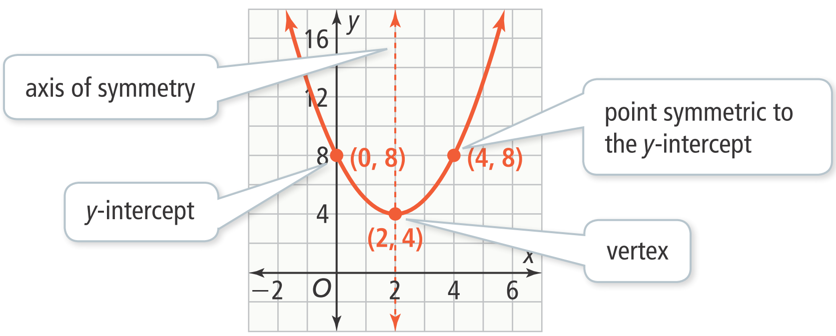
If the y-intercept is the same as the vertex, choose a different point here.

Step 3 Find a point symmetric to the y-intercept across the axis of symmetry.

Since $(0, 8)$ is a point on the parabola 2 units to the left of the axis of symmetry, $x = 2$, $(4, 8)$ will be a point on the parabola 2 units to the right of the axis of symmetry.

Step 4 Sketch the graph.

Once you have three points associated with the quadratic function, you can sketch the parabola based on your knowledge of its general shape.

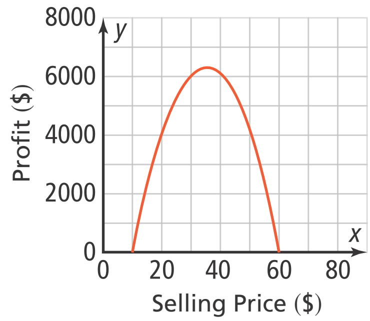


Try It! **2.** Use the key features to graph the function $f(x) = x^2 - 6x - 1$.

APPLICATION

EXAMPLE 3 Interpret the Graph of a Quadratic Function

The graph of the function $f(x) = -10x^2 + 700x - 6{,}000$ shows the profit a company earns for selling headphones at different prices. What is the maximum profit the company can expect to earn?



Formulate The x-axis shows selling price and the y-axis shows the profit. The maximum y-value of the profit function occurs at the vertex of its parabola. Find the vertex of the parabola.

Compute Use the function to find the x- and y-coordinates of the vertex.

Find the x-coordinate of the vertex.

$h = -\frac{b}{2a}$ ······ Use the formula to find the x-coordinate of the vertex.

$h = -\frac{700}{2(-10)}$ ······ Substitute –10 for a and 700 for b.

$h = 35$ ······ Simplify.

> **COMMON ERROR**
> Be careful with the negative signs; there is a negative in the formula and a negative value for a.

Find the y-coordinate of the vertex.

$y = -10x^2 + 700x - 6{,}000$ ······ Write the original function.

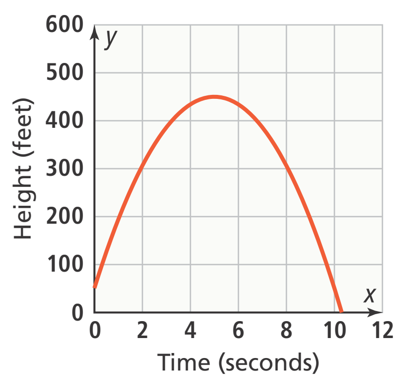
$y = -10(35)^2 + 700(35) - 6{,}000$ ······ Substitute 35 for x.

$y = 6{,}250$ ······ Simplify.

The vertex is (35, 6,250).

Interpret The selling price of \$35 per item gives the maximum profit of \$6,250.

Try It! **3.** A water balloon was thrown from a window. The height of the water balloon over time can be modeled by the function $y = -16x^2 + 160x + 50$. What was the maximum height of the water balloon after it was thrown?



EXAMPLE 4 Write the Equation of a Parabola Given Three Points

What is the equation of a parabola that passes through the points (–2, 32), (1, 5), and (3, 17)?

Step 1 Write three equations by substituting the given *x*- and *y*-values into the standard form of a parabola equation, $y = ax^2 + bx + c$.

$(-2, 32) \quad 32 = a(-2)^2 + b(-2) + c$

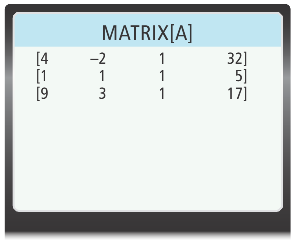
$(1, 5) \quad 5 = a(1)^2 + b(1) + c$
$\quad 5 = 1a + 1b + c$

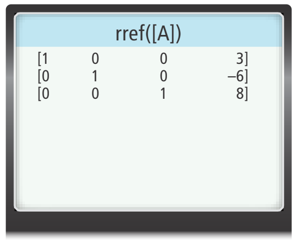
$(3, 17) \quad 17 = a(3)^2 + b(3) + c$
$\quad 17 = 9a + 3b + c$

$$\begin{cases} 32 = 4a - 2b + c \\ 5 = 1a + 1b + c \\ 17 = 9a + 3b + c \end{cases}$$

STUDY TIP
You can also solve a linear system of three equations in three variables by hand, using elimination or substitution.

Step 2 Solve the system.



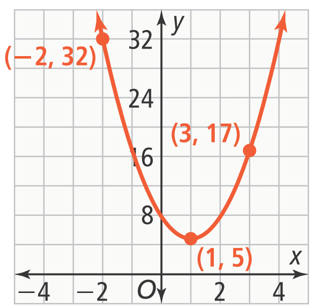


So the solution to the system is $a = 3$, $b = -6$, and $c = 8$.

Step 3 Substitute 3 for *a*, –6 for *b*, and 8 for *c* in the standard form of a quadratic equation.

$y = 3x^2 - 6x + 8$

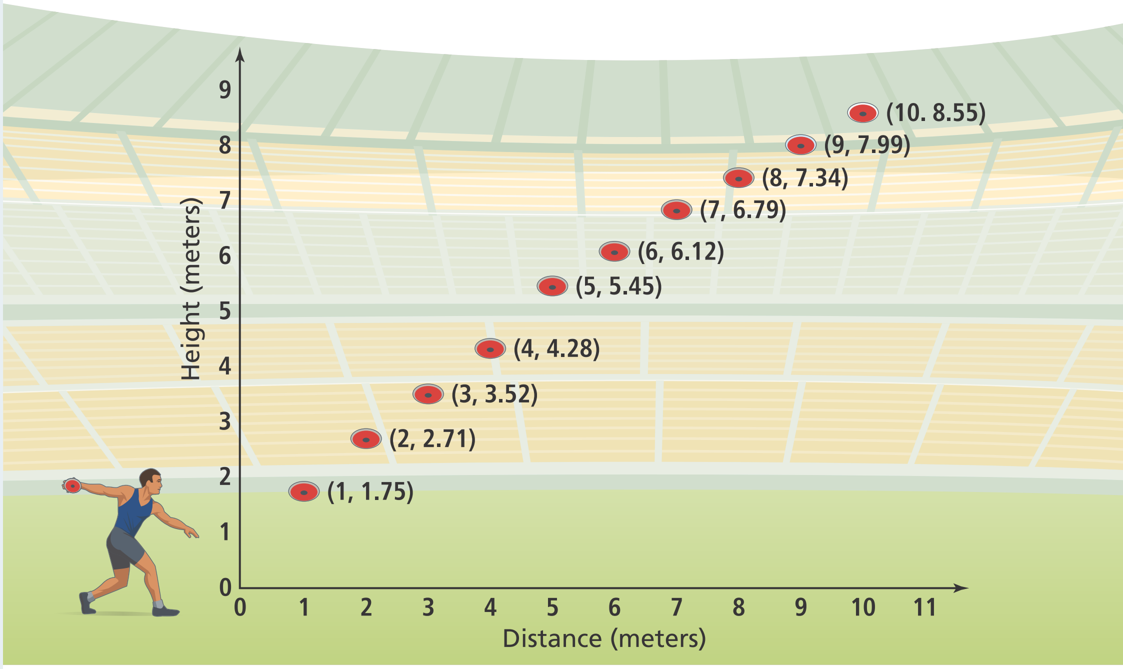
Step 4 Confirm that the graph of the equation passes through the three given points.



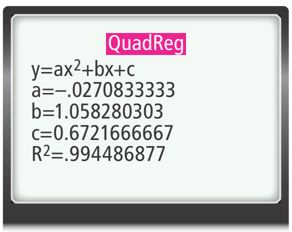
Try It! **4.** What is the equation of a parabola that passes through the points (2, –12), (–1, –15), and (–4, –90)?

EXAMPLE 5 Use Quadratic Regression

Esteban is training for the discus throw. His coach recorded the horizontal distance and height of one of Esteban's discus throws. The graph shows the horizontal distance the discus traveled, in meters, and the height of the discus, in meters. What will be the height of the discus when it has traveled 15 meters from Esteban?



Use graphing technology to perform quadratic regression with the data.



The data show the discus only rising, but the model will resemble a parabola as the discus returns to the ground.

COMMON ERROR
Remember that once you use a regression equation, you are approximating values. Regression is used to make predictions, not to find exact values of variables.

$y \approx -0.027x^2 + 1.058x + 0.672$ .... Write the regression model.

$y \approx -0.027(15)^2 + 1.058(15) + 0.672$ .... Substitute 15 for x.

$y \approx 10.467$ .... Simplify.

Based on this model, when the discus is 15 meters away from Esteban, it will be at a height of approximately 10.5 meters.

Try It!

5. A fan threw a souvenir football into the air from the top of the bleachers toward the bottom of the bleachers. The table shows the height of the football, in feet, above the ground at various times, in seconds. If the football was not touched by anyone on its way to the ground, about how long did it take the football to reach the ground after it was thrown?

Time (s)	0	0.2	0.4	0.6	0.8	1.0
Height (ft)	10	11.76	12.24	11.44	9.36	6.0

CONCEPT SUMMARY Standard Form of a Quadratic Function

STANDARD FORM $y = ax^2 + bx + c$ $\qquad$ $y = -2x^2 - 8x + 1$

KEY FEATURES

Vertex x-coordinate of vertex: $h = -\frac{b}{2a}$ $\qquad$ $h = -\frac{(-8)}{2(-2)} = -2$

Substitute h for x and solve for y to find the y-coordinate of the vertex.

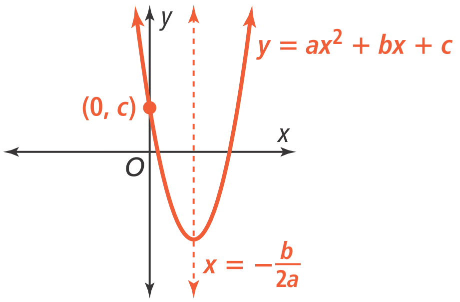
$y = -2(-2)^2 - 8(-2) + 1$
$= -8 + 16 + 1$
$= 9$

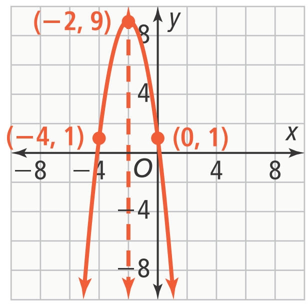
The vertex is (–2, 9).

Axis of Symmetry $x = -\frac{b}{2a}$ $\qquad$ $x = -2$

***y*-intercept** $(0, c)$ $\qquad$ $(0, 1)$

GRAPHS





Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What key features can you determine about a quadratic function from an equation in standard form?

2. **Error Analysis** Cameron said that the y-intercept of a quadratic function always tells the maximum value of that function. Explain Cameron's error.

3. **Vocabulary** Write a quadratic function in standard form.

4. **Make Sense and Persevere** Why do you need at least three points to graph a quadratic function when not given an equation?

Do You KNOW HOW?

Find the vertex and *y*-intercept of the quadratic function.

5. $y = 3x^2 - 12x + 40$ $\qquad$ 6. $y = -x^2 + 4x + 7$

For 7 and 8, find the maximum or minimum of the parabola.

7. $y = -2x^2 - 16x + 20$ $\qquad$ 8. $y = x^2 + 12x - 15$

9. Find the equation in standard form of the parabola that passes through the points (0, 6), (–3, 15), and (–6, 6).

Graph the parabola.

10. $y = 3x^2 + 6x - 2$

11. $y = -2x^2 + 4x + 1$

PRACTICE & PROBLEM SOLVING

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Practice Tutorial

Additional Exercises Available Online

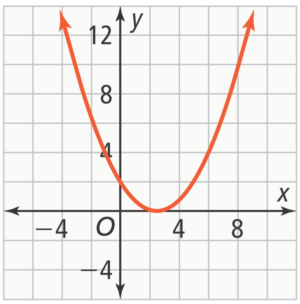
UNDERSTAND

12. **Construct Arguments** Devin found the parabola that fits the three points in the table to be $y = 0.345x^2 - 0.57x - 2.78$. Is Devin correct? Explain.

x	-4	0.6	9
y	5	-3	20

13. **Generalize** How can you find the maximum or minimum value of a quadratic function?

14. **Higher Order Thinking** The quadratic function whose graph is shown represents a cereal bowl. Its equation is $y = 0.32x^2 - 1.6x + 2$. Describe how you could use the function to find the diameter of the cereal bowl if you know its depth.



15. **Error Analysis** Micah found the vertex for the function $y = -9.5x^2 - 47.5x + 63$ as shown.

$x = -\frac{b}{2a}$

$x = -\frac{47.5}{2(-9.5)}$

$x = -\frac{47.5}{-19}$

$x = -(-2.5)$

$x = 2.5$

$y = -9.5(2.5)^2 - 47.5(2.5) + 63$

$y = -59.375 - 118.75 + 63$

$y = -115.125$

Find and correct Micah's error.

PRACTICE

Find the vertex of each parabola. SEE EXAMPLE 1

16. $y = -x^2 + 6x + 30$

17. $y = 3x^2 + 12x - 5$

Find the vertex and y-intercept of the quadratic function, and use them to graph the function. SEE EXAMPLES 1 AND 2

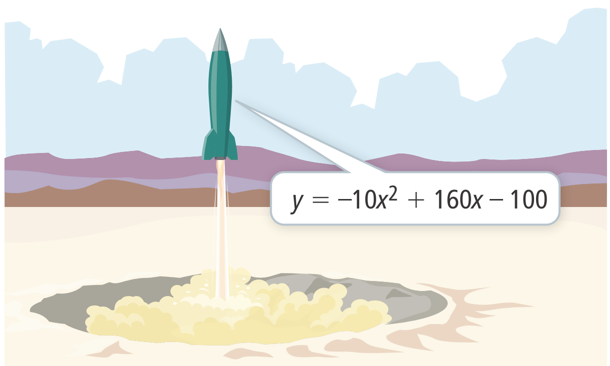
18. $y = -x^2 + 6x - 8$

19. $y = x^2 - 8x + 11$

20. $y = 3x^2 + 18x + 10$

21. $y = -2x^2 - 12x - 5$

22. A rocket is launched into the air. The path of the rocket is modeled by the equation $y = -10x^2 + 160x - 100$. What is the maximum height reached by the rocket, in feet?
SEE EXAMPLE 3



Write the equation of a quadratic function in standard form for the parabola that passes through the given points. SEE EXAMPLE 4

23. $(-1, 5), (4, 0), (5, -7)$

24. $(-2, 2), (1, 8), (4, 50)$

Use quadratic regression to find the equation of a quadratic function that fits the given points.
SEE EXAMPLE 5

25.

x	0	0.5	1	1.5	2
y	35	36	29	14	-9

PRACTICE & PROBLEM SOLVING

APPLY

26. **Model With Mathematics** The height of Amelia's mid-section was measured three times during a long jump.

Time in seconds, x	0	0.5	1
Height in meters, y	0.7	1.5	0.55

Write the equation of a quadratic function that describes Amelia's height as a function of time.

27. **Make Sense and Persevere** A college's business office found the relationship between the number of admissions counselors they employ and the college's profit from tuition could be modeled by the function $y = -10x^2 + 1{,}500x - 35{,}000$.

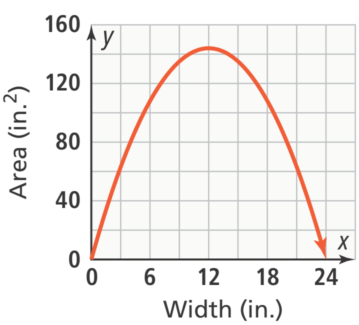
a. Graph the function.

b. How many admissions counselors should the college employ to maximize its profit?

c. What is the maximum amount of profit the college can make?

28. **Mathematical Connections** A rectangular tile has a perimeter of 48 inches.

a. The graph shows the relationship between the **width** of the tile and the **area** of the tile. What function describes this relationship?



b. What is the maximum area? What length and width give the maximum area?

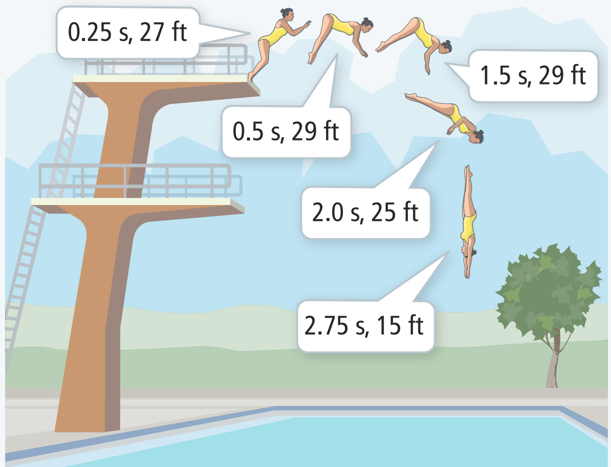
ASSESSMENT PRACTICE

29. The height above the surface of the Earth (in meters) of a rock thrown into the air at 10 m/s after x seconds is given by $f(x) = -9.8\,x^2 + 10x + 1.5$. On the surface of the moon, the height is given by $g(x) = -1.6x^2 + 10x + 1.5$. How much higher does the rock travel on the moon than on Earth? F-IF.2.4

30. **SAT/ACT** Which quadratic equation contains the three points (–4, 12), (2, 42), and (3, 40)?

Ⓐ $y = -x^2 + 3x + 42$
Ⓑ $y = 1.7x^2 - 10x - 55.2$
Ⓒ $y = -1.7x^2 + 10x + 55.2$
Ⓓ $y = x^2 - 3x - 40$
Ⓔ $y = -x^2 + 3x + 40$

31. **Performance Task** A diver jumped from a diving platform. The image shows her height above the water at several different times after leaving the platform.



Part A Find the equation of the quadratic function that describes the relationship between the diver's time and height. Round to the nearest tenth.

Part B How high is the platform the diver jumped from? What is the maximum height reached?

Part C From the maximum height, how long does it take the diver to get halfway down? Which part of the dive is faster, from the top to the halfway point, or from the halfway point to the water? Explain.