Do Now (3/6 & 3/9)

Geometric Sequences...

Geometric Sequences and Series
Section 6.7

Warm-up: Find the next three terms of the sequence.

1. 2, 4, 8, 16, 32, 64, 128 — Geometric Seq.
   ($\cdot 2$)
2. $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, -\frac{1}{243}, \frac{1}{729}, -\frac{1}{2187}$ — Geometric Seq.
   ($\cdot -\frac{1}{3}$)
3. 1, 4, 9, 16, 25, 36, 49 — Neither
   (5^2, 6^2, 7^2)
4. 12, 36, 108, 324, 972, 2916 — Geometric
   ($\cdot 3$)

Definition of a Geometric Sequence

A sequence is geometric if there is a constant ratio between back-to-back numbers.

This ratio is called common ratio r.

Example 1: Write the first five terms of the geometric sequence whose first term is 3 and whose ratio is 2.

3, 6, 12, 24, 48, . . .
($\cdot 2$)

The *n*th Term of a Geometric Sequence

Explicit Formula: $\boxed{a_n = a_1 \cdot r^{n-1}}$

\# of terms
r common ratio
1st term

Recursive Form

$$a_n = \begin{cases} a_1, & n=1 \\ r \cdot a_{n-1}, & n>1 \end{cases}$$

Example 2: Find the 15th term of the geometric sequence whose first term is 20 and whose common ratio is 1.05.

$a_{15} = ?$ $n=15$ $a_1 = 20$ $r = 1.05$

$$a_{15} = 20 \cdot (1.05)^{15-1} = 20(1.05)^{14} \approx \boxed{39.6}$$

Practice Problem 1: Find the ninth term of the geometric sequence whose first term is 4 and whose common ratio is ½.

$$a_9 = 4\left(\tfrac{1}{2}\right)^8 \approx 0.016$$

Example 3: Find a formula for the *n*th term of the following geometric sequence. What is the ninth term?

5, 15, 45, …

$r = 3 \quad a_1 = 5$

$\boxed{a_n = 5 \cdot 3^{n-1}}$

$a_9 = 5 \cdot 3^{9-1}$
$= 5 \cdot 3^8 = 32{,}805$

Practice Problem 2: Find a formula for the *n*th term of the following geometric sequence. What is the tenth term?

6, -2, $\frac{2}{3}$, …

$r = -\frac{2}{6} = -\frac{1}{3}$ or -0.33

$\boxed{a_n = 6 \cdot \left(-\frac{1}{3}\right)^{n-1}}$

$a_{10} = 6\left(-\frac{1}{3}\right)^{10-1} \approx 0.0003$

When you know *any* two terms of a geometric sequence, you can use that information to find a formula for the nth term of the sequence.

Example 4: The fourth term of a geometric sequence is 125, and the 10th term is $\frac{125}{64}$. Find the 14th term.

$a_4 = 125, \quad a_{10} = \frac{125}{64}, \quad a_{14} = ?$

$125 = a_1 \cdot r^3$

$\frac{125}{64} = a_1 \cdot r^9$

LHS: $\cancel{125} \cdot \frac{64}{\cancel{125}} = 64$

RHS: $\frac{1}{r^6}$

$64 \cdot r^6 = \frac{1}{r^6}$

$\frac{64}{64} r^6 = \frac{1}{64}$

$\sqrt[6]{r^6} = \sqrt[6]{\frac{1}{64}}$

$r = \frac{1}{2}$

$125 = a_1\left(\frac{1}{2}\right)^3$

$8 \cdot 125 = a_1\left(\frac{1}{8}\right) \cdot 8$

$a_1 = 1000$

$a_{14} = 1000 \cdot \left(\frac{1}{2}\right)^{14-1}$
$= 0.122$

~~**Practice Problem 3:** The second term of a geometric sequence is -18, and the fifth term is $\frac{2}{3}$. Find the sixth term.~~

$3 + 9 + 27 + \ldots$

The Sum of a Finite Geometric Sequence

Formula: $S_n = \dfrac{a_1(1-r^n)}{1-r}$ ← Geometric Series

Sigma Notation

① $\sum_{m=0}^{n} a_1 \cdot r^m$

② $\sum_{m=1}^{n} a_1 \cdot r^{m-1}$

Example 5:

12   12

(b) $\sum_{n=0}^{12} 5(2)^n$

$a_1 = 5, \; r = 2, \; n = 12$

$S_{12} = \dfrac{5(1-2^{12})}{(1-2)} = \boxed{20{,}475}$

Practice Problem 4: Find the sum: $\sum_{n=0}^{15} 2\left(\frac{4}{3}\right)^n$

$S_{15} = \dfrac{2\left(1-\left(\frac{4}{3}\right)^{15}\right)}{\left(1-\frac{4}{3}\right)}$

The Sum of an Infinite Geometric Series

Formula:

$$S_{12} = \frac{\text{[illegible]}}{(1-2)} = \boxed{20{,}475}$$

Practice Problem 4: Find the sum: $\sum_{n=0}^{15} 2\left(\frac{4}{3}\right)^n$

$$S_{15} = \frac{2\left(1-\left(\frac{4}{3}\right)^{15}\right)}{\left(1-\frac{4}{3}\right)}$$

The Sum of an Infinite Geometric Series

Formula:

Example 6: Find each sum

a) $\sum_{n=0}^{\infty} 4(0.6)^n$

b) $\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n$

c) $\sum_{n=0}^{\infty} 2(3)^n$