Activity Assess

2-3 Factored Form of a Quadratic Function

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I CAN… find the zeros of quadratic functions.

VOCABULARY

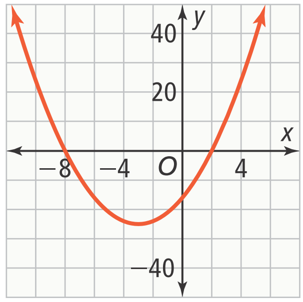
- Zero Product Property

MAFS.912.A-SSE.2.3.a–Factor a quadratic expression to reveal the zeros of the function it defines. Also A-SSE.1.2, A-APR.2.3

MAFS.K12.MP.1.1, MP.3.1, MP.7.1

CRITIQUE & EXPLAIN

Corey wrote an equation in factored form, $y = (x + 8)(x - 2)$, to represent a quadratic function. Kimberly wrote the equation $y = x^2 + 6x - 16$, and Joshua wrote the equation $y = (x + 3)^2 - 25$.



A. Reason Do all three equations represent the same function? If not, whose is different? Explain algebraically.

B. How else could you determine if all three equations represent the same function?

C. What information can Corey's form help you find that is more difficult to find using Kimberly's or Joshua's form?

ESSENTIAL QUESTION

How is the factored form helpful in solving quadratic equations?

EXAMPLE 1 Factor a Quadratic Expression

Factor the expression.

A. $x^2 + 7x + 12$

Recall that using the Distributive Property, $(x + m)(x + n) = x^2 + (m + n)x + mn$.

$$x^2 + \underbrace{7}_{m+n}x + \underbrace{12}_{mn}$$

Add factor pairs of 12 to find the numbers that add to 7. The numbers 3 and 4 have a product of 12 and a sum of 7. Therefore, the factored form of the expression $x^2 + 7x + 12$ is $(x + 3)(x + 4)$.

B. $2x^2 - 5x - 3$

When the leading coefficient is not 1, multiply the leading coefficient and the constant. Look for factors of this product that add to the middle coefficient. Rewrite the middle term using these factors, then factor by grouping.

$2x^2 - 5x - 3$ The factors of -6 that have a sum of -5 are 1 and -6.

$2x^2 + x - 6x - 3$ Rewrite $-5x$ as $x - 6x$.

$x(2x + 1) - 3(2x + 1)$

$(2x + 1)(x - 3)$

The factored form of the expression $2x^2 - 5x - 3$ is $(2x + 1)(x - 3)$.

STUDY TIP

You can check your work by multiplying the factors using the Distributive Property.

Try It!

1. Factor the expression.

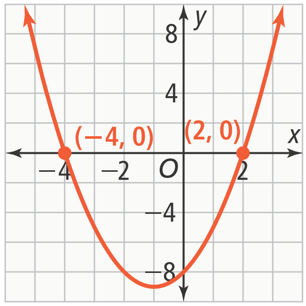
a. $x^2 - 9$ **b.** $3x^2 - 7x + 2$

Activity Assess

CONCEPTUAL UNDERSTANDING

EXAMPLE 2 Relate Factors to Zeros of a Function

The graph shows the function defined by $y = x^2 + 2x - 8$. How do the zeros of the function relate to the factors of the expression $x^2 + 2x - 8$?



The expression $x^2 + 2x - 8$ can be represented as a product of two factors. The factors of −8 that have a sum of 2 are 4 and −2.

$$y = x^2 + 2x - 8 \quad \rightarrow \quad y = (x + 4)(x - 2)$$

The x-intercepts of the graph are −4 and 2, so the zeros of the function are $x = -4$ and $x = 2$.

Substitute $x = -4$ and $x = 2$ in to the factored form of the equation.

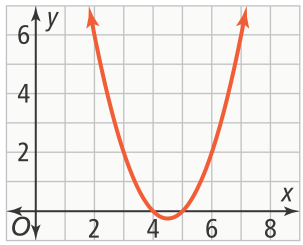
$$y = (-4 + 4)(-4 - 2) = 0(-6) = 0$$

$$y = (2 + 4)(2 - 2) = 6(0) = 0$$

The factors, $(x + 4)$ and $(x - 2)$, are related to the zeros $x = -4$ and $x = 2$ since each of the zeros makes one of the factors 0.

VOCABULARY
Recall that a zero of a function is a number, z, for which $f(z) = 0$. A zero of f is also called an x-intercept of the graph since the graph of f passes through the point $(z, 0)$.

Try It! 2. The graph shows the function $y = x^2 - 9x + 20$. Identify the zeros of the function. How do the zeros relate to the factors of $x^2 - 9x + 20$?



CONCEPT Zero Product Property

The **Zero Product Property** states that if a product of real-number factors is 0, then at least one of the factors must be 0.

In the case of two factors, if $ab = 0$, then either $a = 0$ or $b = 0$, or both.

To use the Zero Product Property, rewrite the equation so that it is an expression equal to 0, then factor and solve.

EXAMPLE 3 Solve Quadratic Equations by Factoring

Solve the equation.

A. $x^2 + x = 42$

$x^2 + x - 42 = 0$	Set equation equal to 0.
$(x + 7)(x - 6) = 0$	Factor.
$x + 7 = 0$ or $x - 6 = 0$	Use the Zero Product Property.
$x = -7$ or $x = 6$	Solve.

MAKE SENSE AND PERSEVERE
If you can write an expression in factored form, you can find the value of the variable that makes each factor 0. These values are the zeros of the function.

CONTINUED ON THE NEXT PAGE

EXAMPLE 3 CONTINUED

B. $2x^2 = -9x + 5$

$2x^2 + 9x - 5 = 0$ ······ Rewrite as equation equal to 0.

$2x^2 - x + 10x - 5 = 0$ ······ Factor by grouping.

$x(2x - 1) + 5(2x - 1) = 0$

$(x + 5)(2x - 1) = 0$

$x + 5 = 0$ or $2x - 1 = 0$ ······ Use the Zero Product Property.

$x = -5$ or $x = \frac{1}{2}$ ······ Solve.

STUDY TIP

Check your work algebraically, by plugging the solutions in to the original equation. Or check graphically by confirming that your solutions are the x-intercepts of the graph.

Try It! **3.** Solve the equation by factoring.

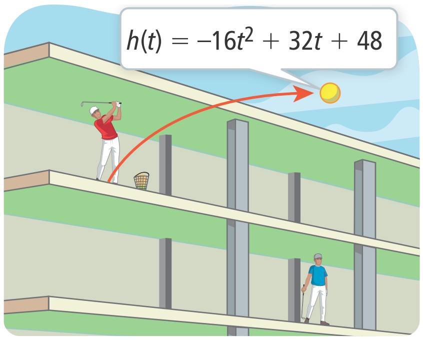
a. $x^2 + 8x = 20$

b. $2x^2 = 3x + 2$

APPLICATION

EXAMPLE 4 Find the Zeros of a Quadratic Function

Marco hits a golf ball into the air from the third level of a driving range that is 48 ft high. The function $h(t) = -16t^2 + 32t + 48$ gives the height, h, in feet, of the golf ball t seconds after it is hit into the air. When will the ball hit the ground?



The ball hits the ground when the height, $h(t)$, is 0.

$0 = -16t^2 + 32t + 48$ ······ Substitute 0 for $h(t)$.

$0 = -16(t^2 - 2t - 3)$ ······ Factor out the GCF, -16.

$0 = -16(t + 1)(t - 3)$ ······ Factor.

$t + 1 = 0$ or $t - 3 = 0$ ······ Use the zero Product Property.

$t = -1$ or $t = 3$ ······ Solve.

The zeros of the function are at $t = -1$ and $t = 3$.

Since time has to be positive, $t = 3$ is the only solution that makes sense.

This means that after 3 seconds, the golf ball will hit the ground.

COMMUNICATE PRECISELY

Zeros of the function $h(t) = -16t^2 + 32t + 48$ are solutions of the equation $0 = -16t^2 + 32t + 48$.

Try It! **4.** A baseball is thrown from the upper deck of a stadium, 128 ft above the ground. The function $h(t) = -16t^2 + 32t + 128$ gives the height of the ball t seconds after it is thrown. How long will it take the ball to reach the ground?

EXAMPLE 5 Determine Positive or Negative Intervals

Identify the interval(s) on which the function $y = x^2 - 2x - 3$ is positive.

The *y*-values of a quadratic function can only turn from positive to negative or from negative to positive when the graph crosses the *x*-axis. Find the zeros of the function to identify these points.

$0 = x^2 - 2x - 3$	Set expression equal to 0.
$0 = (x - 3)(x + 1)$	Factor.
$x - 3 = 0$ or $x + 1 = 0$	Zero Product Property
$x = 3$ or $x = -1$	Solve.

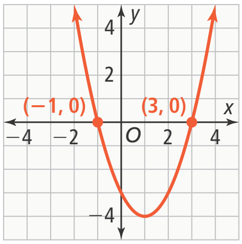
Two zeros create three intervals. Choose an *x*-value to test in each interval. Substitute the *x*-value into the original expression to determine if the corresponding *y*-value is positive or negative.

LOOK FOR RELATIONSHIPS
The sign of the *y*-value of the test point is the same as for the *y*-value of any other point over the entire interval you are testing.

$x < -1$	$-1 < x < 3$	$x > 3$
Choose $x = -3$. $(-3)^2 - 2(-3) - 3$ $= 9 + 6 - 3$ $= 12$	Choose $x = 1$. $(1)^2 - 2(1) - 3$ $= 1 - 2 - 3$ $= -4$	Choose $x = 6$. $(6)^2 - 2(6) - 3$ $= 36 - 12 - 3$ $= 21$
Positive	Negative	Positive

Graph the function to verify where the function is positive or negative.

The function is positive when the graph is above the *x*-axis, or on the intervals $x < -1$ and $x > 3$.



Try It! 5. Identify the interval(s) on which the function $y = x^2 - 4x - 21$ is negative.

EXAMPLE 6 Write the Equation of a Parabola in Factored Form

Write an equation of a parabola with *x*-intercepts at (–2, 0) and (–1, 0) and which passes through the point (–3, 20).

$y = a(x - p)(x - q)$	Write the general form of a factored equation.
$y = a(x - (-2))(x - (-1))$	Substitute –1 and –2 for zeros.
$y = a(x + 2)(x + 1)$	Simplify.
$20 = a(-3 + 2)(-3 + 1)$	Substitute –3 for *x* and 20 for *y*.
$20 = 2a$	Simplify.
$10 = a$	Solve.
$y = 10(x + 2)(x + 1)$	Subsitute 10 for *a*.

COMMON ERROR
If $x = -2$ is an *x*-intercept, then $x + 2$ is the factor, not $x - 2$.

Try It! 6. Write an equation of a parabola with *x*-intercepts at (3, 0) and (–3, 0) and which passes through the point (1, 2).

CONCEPT SUMMARY Factored Form of a Quadratic Function

FACTORED FORM $y = ax^2 + bx + c$ can be written as $0 = a(x - p)(x - q)$, where p and q are the zeros of the function. The x-intercepts of the graph correspond to the zeros of the function. Two zeros denote 3 intervals of x values.

GRAPH For the function $y = 2x^2 + 3x - 14$, write the equation $0 = 2x^2 + 3x - 14$ in factored form to identify the zeros.

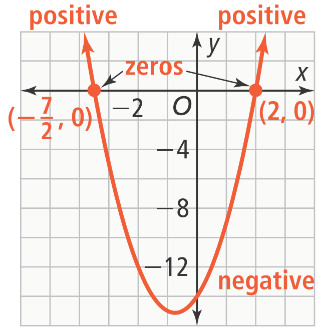
$0 = 2x^2 + 3x - 14$

$0 = (2x + 7)(x - 2)$

The zeros of the function are $x = -\frac{7}{2}$ and $x = 2$.

intervals where function values are positive:
$x < -\frac{7}{2}$, and $x > 2$

interval where function values are negative: $-\frac{7}{2} < x < 2$



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How is the factored form helpful in solving quadratic equations?

2. **Error Analysis** Amir says the graph of $y = x^2 + 16$ has −4 as a zero. Is Amir correct? Explain.

3. **Vocabulary** How does the factored form of a quadratic equation relate to the Zero Product Property?

4. **Generalize** How does knowing the zeros of a function help determine where a function is positive?

Do You KNOW HOW?

Factor each expression.

5. $x^2 - 5x - 24$

6. $5x^2 + 3x - 2$

Solve each equation.

7. $x^2 = 12x - 20$

8. $4x^2 - 5x = 6$

9. The height, in feet, of a t-shirt launched from a t-shirt cannon high in the stands at a football stadium is given by $h(x) = -16x^2 + 64x + 80$, where x is the time in seconds after the t-shirt is launched. How long will it take before the t-shirt reaches the ground?

PRACTICE & PROBLEM SOLVING

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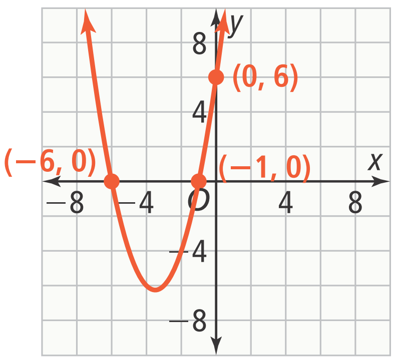
UNDERSTAND

10. Generalize Can you write the equation of a quadratic function knowing its zeros and its non-zero y-intercept? If so, describe the process. If not, explain why.

11. Error Analysis Describe and correct the error a student made in solving a quadratic equation.

$$0 = 2x^2 + 7x + 5$$
$$0 = 2x^2 + 2x + 5x + 5$$
$$0 = 2x(x + 1) + 5(x + 1)$$
$$0 = 2x,\ 0 = x + 1,\ 0 \neq 5$$
$$0 = x,\ -1 = x$$

12. Model With Mathematics Use the graph of the function to write the equation in factored form.



13. Generalize For what values of x is the expression $(x - 4)^2 > 0$?

14. Error Analysis A student says that the zeros of $y = (x - 2)(x + 7)$ are -2 and 7. Is the student correct? If not, describe and correct the error the student made.

15. Construct Arguments Explain why $x^2 + 25$ is not equal to $(x + 5)^2$.

16. Mathematical Connections Describe how factoring can help you find the x-intercepts of the graph of the quadratic function $y = x^2 - 4x + 3$.

PRACTICE

Factor each quadratic expression. SEE EXAMPLE 1

17. $x^2 - 3x - 10$

18. $3x^2 - 5x - 12$

19. $x^2 + 15x + 56$

20. $2x^2 + 7x - 15$

21. $3x^2 - 18x - 48$

22. $4x^2 - 11x - 3$

23. What are the zeros of the quadratic function $y = 3(x - 5)(x + 4)$? SEE EXAMPLE 2

Solve each quadratic equation. SEE EXAMPLE 3

24. $x^2 - 5x - 14 = 0$

25. $x^2 = 5x - 6$

26. $3x^2 - 60 = 3x$

27. $5x^2 + 12x = 9$

28. $4x^2 + 3x - 7 = 0$

29. $6x^2 = 5x + 6$

30. A penny is dropped from the top of a new building. Its height in feet can be modeled by the equation $y = 256 - 16x^2$, where x is the time in seconds since the penny was dropped. How long does it take for the penny to reach the ground? SEE EXAMPLE 4

Identify the interval(s) on which each quadratic function is positive. SEE EXAMPLE 5

31. $y = x^2 + 9x + 18$

32. $y = x^2 + 2x - 8$

33. $y = x^2 - 5x - 24$

34. $y = -x^2 + 4x + 12$

35. $y = 2x^2 + 12x + 18$

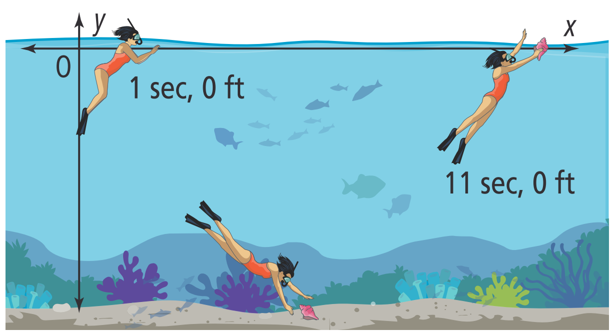
36. $y = 5x^2 - 3x - 8$

Write an equation for each parabola. SEE EXAMPLE 6

37. A parabola with x-intercepts at $(-1, 0)$ and $(3, 0)$ which passes through the point $(1, -8)$

38. A parabola with x-intercepts at 0 and 1 and which passes through the point $(2, -2)$

39. A snorkeler dives for a shell on a reef. After entering the water, the diver descends $\frac{11}{3}$ ft in one second. Write an equation that models the diver's position with respect to time.

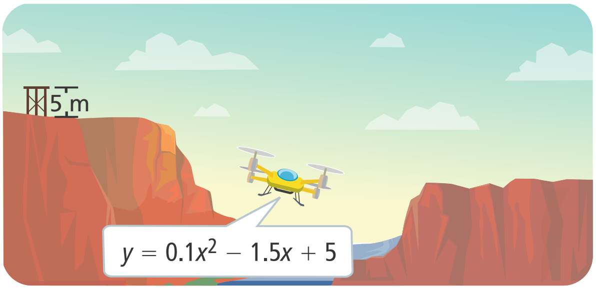


PRACTICE & PROBLEM SOLVING

APPLY

40. **Make Sense and Persevere** Rectangular apartments are 12 ft longer than they are wide. Each apartment has 1,053 ft^2 of floor space. What are the dimensions of an apartment? Explain.

41. **Use Structure** The height of a drone, in meters, above its launching platform that is 5 m above the ground, is modeled by $y = 0.1x^2 - 1.5x + 5$, where x is the time in seconds. The drone leaves the launch pad, flies down into a canyon, and then it flies back up again.



a. What is the factored form of the equation for the height of the drone?

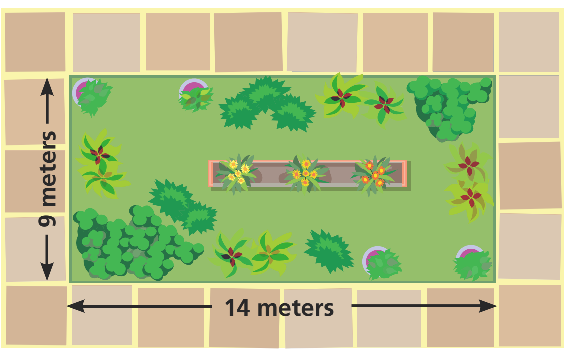
b. After how many seconds will the drone be at ground level?

c. After how many seconds will the drone come back to the height of its platform?

42. **Higher Order Thinking** LaTanya is designing a rectangular garden with a uniform walkway around its border. LaTanya has 140 m^2 of material to build the walkway.

a. Write an equation for the dimensions of the garden and the surrounding walkway.

b. How wide is the walkway? Explain.



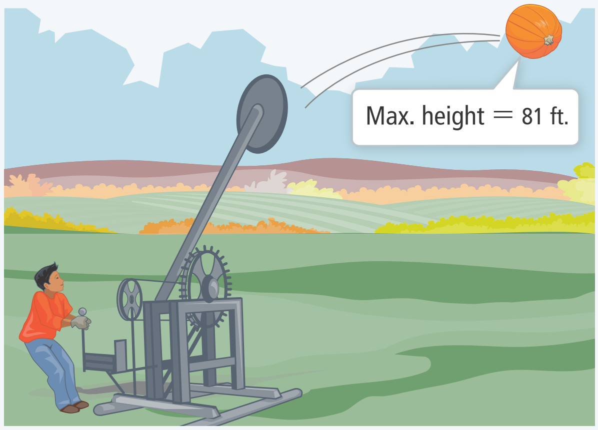
ASSESSMENT PRACTICE

43. A factory can use either of two processes to manufacture a product. The costs, in dollars, for each process are given by the functions $f(x) = 6x$ and $g(x) = x^2 + 5$, where x represents the number of units, in thousands. Solve the equation $f(x) = g(x)$. When are the costs for both processes the same? A-SSE.2.3.a

44. **SAT/ACT** What is the sum of the zeros of the function $y = x^2 - 9x - 10$?

Ⓐ –10 Ⓑ –9 Ⓒ 0 Ⓓ 9 Ⓔ 10

45. **Performance Task** A pumpkin is launched from the ground into the air and lands 4.5 s later.



Part A Write a quadratic function that models the height, in feet, of the pumpkin x seconds after it is launched. Explain how you found the function.

Part B A second pumpkin is launched from the ground. After 1 second, it is 64 feet high. The pumpkin lands after 5 seconds. What is the maximum height of the pumpkin? Explain.