2-1 Vertex Form of a Quadratic Function

PearsonRealize.com

I CAN... identify key features of quadratic functions.

VOCABULARY

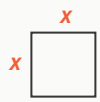
- parabola
- quadratic function
- vertex form of a quadratic function

MAFS.912.F-BF.2.3–Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific value of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
Also A-CED.1.2, F-IF.2.4
MAFS.K12.MP.1.1, MP.3.1, MP.7.1

Activity Assess

EXPLORE & REASON

The table represents $A(x)$, the area of a square as a function of side length x units, where x is a positive real number.

Side Length (units)	x	1	2	3	4
Model					
Area (sq. units)	$A(x)$	1	4	9	16

A. Consider the function where the areas in the table are doubled. Write the equation of a function that represents this.

B. **Look for Relationships** Graph the ordered pairs for both $A(x)$ and your new function. How would you describe the differences in the locations of these points?

C. Find the equation for a function whose x-values are the same as $A(x)$ but whose y-values are 2 units greater than each y-value in $A(x)$.

ESSENTIAL QUESTION

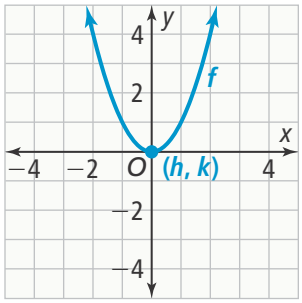
How does the equation of a quadratic function in vertex form highlight key features of the function's graph?

CONCEPT Representations of Quadratic Functions

A function is a **quadratic function** if its equation can be written in the form $f(x) = ax^2 + bx + c$, with $a \neq 0$.

All quadratic functions are transformations of the parent function defined by $f(x) = x^2$.

The graph of a quadratic function is called a **parabola**.



The **vertex form of a quadratic function** is $f(x) = a(x - h)^2 + k$ where (h, k) is the vertex of the parabola. Vertex form is useful because it highlights the vertex of the graph of the quadratic function.

CONCEPTUAL UNDERSTANDING

EXAMPLE 1 Transform a Quadratic Function

How are transformations of the graph of $f(x) = x^2$ related to an equation representing another quadratic function?

Vertex form shows three different ways in which the graph of the function $f(x) = x^2$ may be transformed.

$$f(x) = a(x - h)^2 + k$$

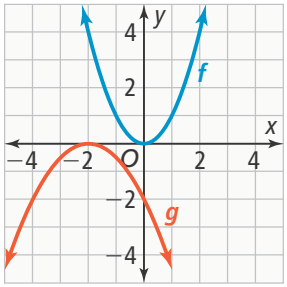
The value of a determines the direction the parabola opens and whether the graph is stretched or compressed.

The value of h determines the horizontal translation.

The value of k determines the vertical translation.

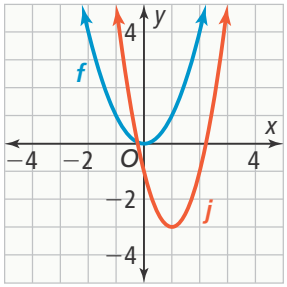
A. $g(x) = -\frac{1}{2}(x + 2)^2$

The equation shows that the graph is to be translated 2 units left, will open downward, and will be vertically compressed.



B. $j(x) = 2(x - 1)^2 - 3$

The equation shows that the graph is to be translated 1 unit right, vertically stretched, and translated down 3 units. It will open upward.



STUDY TIP
Recall that a vertical stretch makes the graph narrower and that a vertical compression makes the graph wider. To see the effects easily, use the same axes or units for all graphs.

When $a > 0$ the parabola opens upward. When $a < 0$, the parabola opens downward. When $|a| > 1$, the graph is stretched, and when $0 < |a| < 1$, the graph is compressed.

Try It! **1.** Describe the transformations of the parent function $f(x) = x^2$. Then graph the function.

a. $g(x) = -(x + 2)^2$ **b.** $g(x) = (x - 1)^2 + 2$

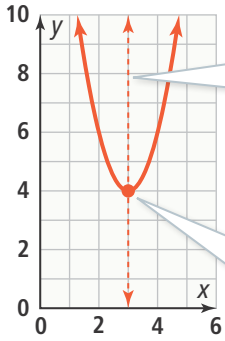
EXAMPLE 2 Determine Key Features of a Quadratic Function

What are the key features of the quadratic function $f(x) = 2(x - 3)^2 + 4$?

The graph represents $f(x) = 2(x - 3)^2 + 4$.

The 2 indicates that the graph opens upward and is vertically stretched.

The range is $y \geq 4$. There are no restrictions on the value of x, so the domain is all real numbers.



The axis of symmetry is the line $x = h$ or $x = 3$.

The vertex is the ordered pair (h, k), or $(3, 4)$.

Try It! **2.** Identify the vertex, axis of symmetry, minimum or maximum, domain, and range of the function $f(x) = -(x + 4)^2 - 5$.

EXAMPLE 3 Write an Equation of a Parabola

What is the equation of a quadratic function with vertex (−2, 3) and *y*-intercept −1?

Step 1 Substitute the coordinates of the vertex for h and k in the vertex form of a quadratic function.

$(h, k) = (-2, 3)$, so $y = a(x - (-2))^2 + 3$

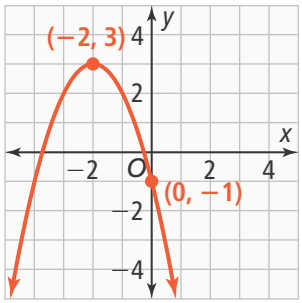
Step 2 Substitute the values of x and y from the y-intercept, and then solve for a.

$(x, y) = (0, -1)$, so $-1 = a(0 + 2)^2 + 3$

$-4 = a(2)^2$

$-4 = 4a$

$a = -1$



Step 3 Substitute the value of a into the vertex form of a quadratic function.

$a = -1$ so $y = -(x + 2)^2 + 3$

The equation of the parabola is $y = -(x + 2)^2 + 3$.

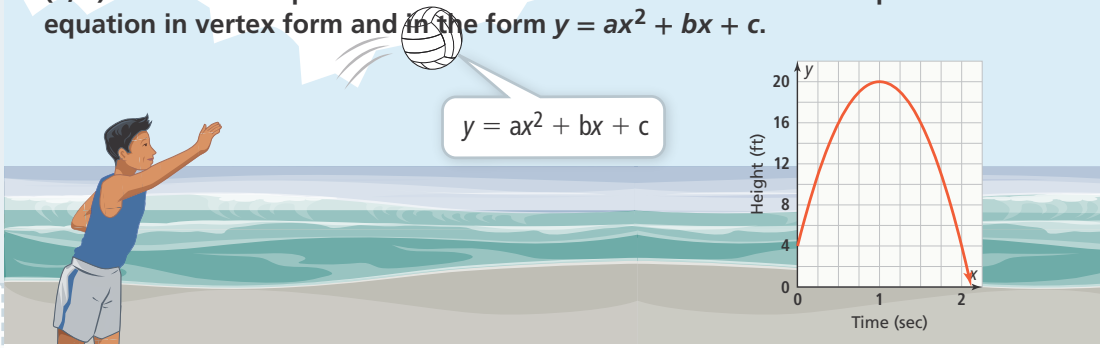
COMMON ERROR
Be careful to not switch the coordinate values when substituting them into the equation.

Try It! **3.** What is the equation of a parabola with a vertex of (1, −4) and which passes through (−2, −1)?

APPLICATION

EXAMPLE 4 Write an Equation of a Parabola Given the Graph

The height of a thrown ball is a quadratic function of the time it has been in the air. The graph of the quadratic function is the parabolic path of the ball. The vertex of the graph is (1, 20) and the path of the ball includes the point (0, 4). What is an expression that defines this function? Write the quadratic equation in vertex form and in the form $y = ax^2 + bx + c$.



$y = a(x - h)^2 + k$

$4 = a(0 - 1)^2 + 20$

$4 = a(-1)^2 + 20$

$4 = a + 20$

$-16 = a$

Find a by substituting the vertex and a given point.

$y = -16(x - 1)^2 + 20$

$y = -16(x^2 - 2x + 1) + 20$

$y = -16x^2 + 32x + 4$

The equation of the parabola in vertex form is $y = -16(x - 1)^2 + 20$.

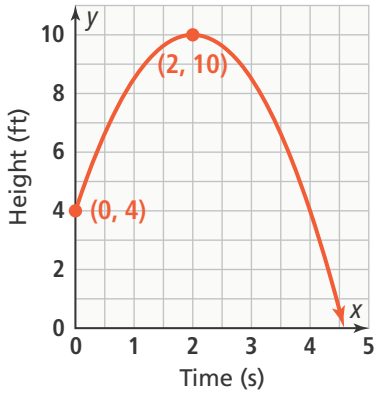
In the form $y = ax^2 + bx + c$, the equation is $y = -16x^2 + 32x + 4$.

LOOK FOR RELATIONSHIPS
By converting vertex form into standard form, you can see how h and k relate to the coefficients of the equation.

CONTINUED ON THE NEXT PAGE

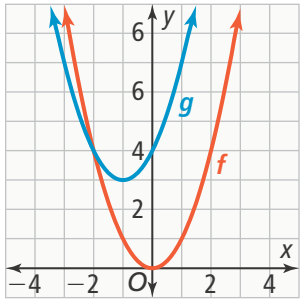
EXAMPLE 4 CONTINUED

Try It! **4.** The graph shows the height of the flying disk with respect to time. What is the equation of the function? Write the equation in vertex form. Then write the equation in the form $y = ax^2 + bx + c$.



EXAMPLE 5 Write an Equation of a Transformed Function

The function *g* is a translation of the parent function *f* 1 unit left and 3 units up. What is the equation of *g*? Write the quadratic equation in vertex form and in the form $f(x) = ax^2 + bx + c$.



Translate the graph of $f(x)$ left 1 unit to locate the graph of $f(x + 1)$, then translate the graph of $f(x + 1)$ up 3 units to locate the graph of $f(x + 1) + 3$.

$g(x) = f(x + 1) + 3$

$g(x) = a(x + 1)^2 + 3$

From the graph, the point (0, 4) appears to be on g. Use the point (0, 4) to find a.

$4 = a(0 + 1)^2 + 3$

$4 = a + 3$

$1 = a$

Substituting $a = 1$, the equation is

$g(x) = a(x + 1)^2 + 3$

$g(x) = (x + 1)^2 + 3$

$g(x) = x^2 + 2x + 1 + 3$

$g(x) = x^2 + 2x + 4$

In vertex form, $g(x) = (x + 1)^2 + 3$ and in the form $y = ax^2 + bx + c$, the equation is $g(x) = x^2 + 2x + 4$.

STUDY TIP
You can confirm your equation by picking a point that is on the graph and checking to make sure it satisfies your equation.

Try It! **5.** What is the equation of j? Write the equation in vertex form and in the form $y = ax^2 + bx + c$.

a. Let j be a quadratic function whose graph is a translation 2 units right and 5 units down of the graph of f.

b. Let j be a quadratic function whose graph is a reflection of the graph of f in the x-axis followed by a translation 1 unit down.

CONCEPT SUMMARY Vertex Form of a Quadratic Function

WORDS The graph of a quadratic function is called a parabola.

A quadratic function can be represented by an equation in vertex form $y = a(x - h)^2 + k$. Vertex form shows the different ways in which the graph of the parent function $f(x) = x^2$ can be transformed.

ALGEBRA

$f(x) = x^2$
vertex (0, 0)
axis of symmetry $x = 0$
opens upward
minimum $y = 0$
domain $(-\infty, \infty)$
range $[0, \infty)$

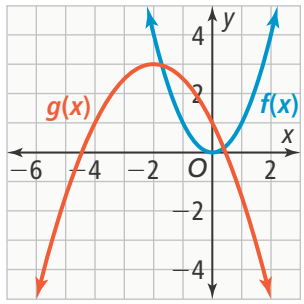
$y = a(x - h)^2 + k$
$a \neq 0$
vertex (h, k)
axis of symmetry $x = h$
domain: $(-\infty, \infty)$

If $a > 0$:
opens upward
minimum $y = k$
range: $[k, \infty)$

if $a < 0$:
opens downward
minimum $y = k$
range: $(-\infty, k]$

NUMBERS $g(x) = -\frac{1}{2}(x + 2)^2 + 3$
vertex $(-2, 3)$
axis of symmetry $x = -2$
opens downward
maximum $y = 3$
domain $(-\infty, \infty)$
range $(-\infty, 3]$

GRAPH



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does the equation of a quadratic function in vertex form highlight key features of the function's graph?

2. **Error Analysis** Given the function $g(x) = (x + 3)^2$, Martin says the graph should be translated right 3 units from the parent graph $f(x) = x^2$. Explain his error.

3. **Vocabulary** What shape does a quadratic function have when graphed?

4. **Communicate Precisely** How are the graphs of $f(x) = x^2$ and $g(x) = -(x + 2)^2 - 4$ related?

Do You KNOW HOW?

Describe the transformation of the parent function $f(x) = x^2$.

5. $g(x) = -(x + 5)^2 + 2$

6. $h(x) = (x + 2)^2 - 7$

Write the equation of each parabola in vertex form.

7. Vertex: (−3, 7); Point: (−2, −5)

8. Vertex: (1, 3); Point: (2, 5)

9. Vertex: (−4, 6); Point: (−2, −2)

10. Vertex: (7, 4); Point: (5, 16)

PRACTICE & PROBLEM SOLVING

Scan for Multimedia

Practice Tutorial

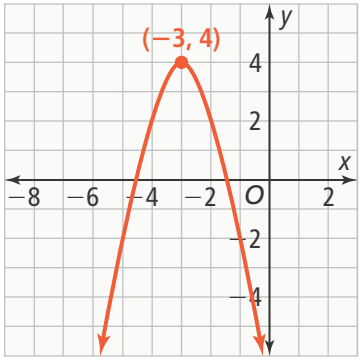
Additional Exercises Available Online

UNDERSTAND

11. Use Structure The graph of the function $f(x) = x^2$ will be translated 3 units up and 1 unit left. What is the resulting function $g(x)$?

12. Error Analysis A classmate said that the vertex of $g(x) = -5(x + 2)^2 - 4$ is (2, 4). Is your classmate correct? If not, what is the correct vertex?

13. Higher Order Thinking The graph below is a transformation of the graph of the parent function. Write the quadratic function to model the graph.



14. Construct Arguments Explain why the graph of the equation $g(x) = -(x + 1)^2 - 3$ would be a parabola opening downward.

15. Use Structure Amaya is standing 30 ft from a volleyball net. The net is 8 ft high. Amaya serves the ball. The path of the ball is modeled by the equation $y = -0.02(x - 18)^2 + 12$, where x is the ball's horizontal distance in feet from Amaya's position and y is the distance in feet from the ground to the ball.

a. How far away is the ball from Amaya when it is at its maximum height? Explain.

b. Describe how you would find the ball's height when it crosses the net at $x = 30$.

PRACTICE

Describe the transformation of the parent function $f(x) = x^2$. Then graph the transformed function. SEE EXAMPLE 1

16. $f(x) = (x - 1)^2 + 3$

17. $y = (x + 1)^2 - 3$

18. $g(x) = 2x^2$

19. $f(x) = -(x - 1)^2 + 7$

20. $y = -2(x + 1)^2 + 1$

21. $f(x) = \frac{1}{2}(x - 2)^2 + 3$

Identify the vertex, axis of symmetry, maximum or minimum, domain, and range of each function. SEE EXAMPLE 2

22. $y = 2(x - 2)^2 + 5$

23. $f(x) = -(x - 1)^2 + 2$

24. $g(x) = -(x + 4)^2$

25. $y = \frac{1}{3}(x + 2)^2 - 1$

Write the equation of each parabola in vertex form. SEE EXAMPLE 3

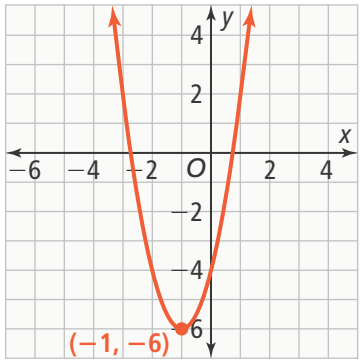
26. Vertex: (1, 2); Point: (2, −5)

27. Vertex: (3, 6); y-intercept: 2

28. Vertex: (0, 5); Point: (1, −2)

Write the equation of the function represented by the parabola in vertex form and in the form $y = ax^2 + bx + c$. SEE EXAMPLE 4

29.



Write the equation $g(x)$ in vertex form of a quadratic function for the transformations given the function $f(x) = x^2$. SEE EXAMPLE 5

30. Let $g(x)$ be the function whose graph is a translation 4 units left and 1 unit up of the graph of $f(x)$.

31. Let $g(x)$ be the function whose graph is a reflection in the x-axis and translated 3 units right of the graph of $f(x)$.

Go Online | PearsonRealize.com

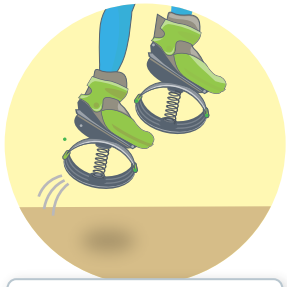
PRACTICE & PROBLEM SOLVING

Practice Tutorial
Mixed Review Available Online

APPLY

32. Look for Relationships The height, in inches, that a person can jump while wearing a pair of jumping shoes is based on the time, x, in seconds, from the start of the jump. Beth is testing out Max Jumps and Jumpsters to determine which shoes she likes better. Compare the maximum heights on the two sets of shoes.

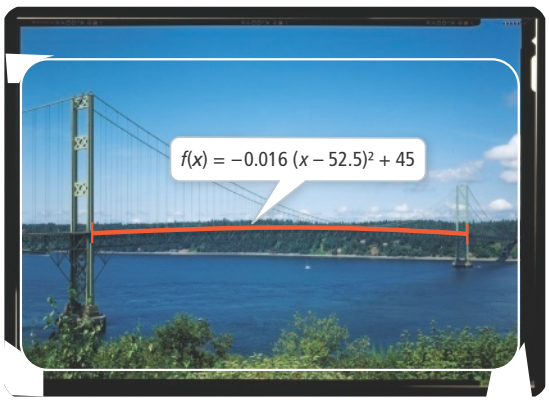
$f(x) = -192(x - 0.289)^2 + 16$



$g(x) = -192(x - 0.445)^2 + 38$

33. Make Sense and Persevere Find three additional points on the parabola that has vertex (1, –2) and passes through (0, –5).

34. Make Sense and Persevere The curvature of the Tacoma Narrows Bridge in Washington is in the shape of a parabola.



In the given function, x represents the horizontal distance (in meters) from the arch's left end and $f(x)$ represents the distance (in meters) from the base of the arch. What is the width of the arch?

35. Model With Mathematics An object is thrown from a height of 5 in. After 2 s, the object reaches a maximum height of 9 in., and then it lands back on the ground 5 s after it was thrown. Write the vertex form of the quadratic equation that models the object's path, and draw the graph.

ASSESSMENT PRACTICE

36. The graph of $g(x) = 3(x - 2)^2$ is a transformation of the graph of $f(x) = x^2$. Describe in words the sequence of transformations that takes the graph of f to the graph of g. F-BF.2.3

37. SAT/ACT Which of the following functions represents a parabola with a vertex at (–3, 4) and that passes through the point (–1, –4)?

Ⓐ $f(x) = x^2 - 5$
Ⓒ $f(x) = 2(x + 1)^2 - 4$
Ⓑ $f(x) = -2(x + 3)^2 + 4$
Ⓓ $f(x) = 2(x - 3)^2 - 32$

38. Performance Task The Bluebird Bakery sells more cookies when it lowers its prices, but this also changes profits.

The profit function for the cookies is $f(x) = -500(x - 0.45)^2 + 400$. This function represents the profit earned when the price of a cookie is x dollars. The bakery wants to maximize their profits.

Part A What is the domain of the function?

Part B Find the daily profits for selling cookies for $0.40 each and for $0.75 each.

Part C What price should the bakery charge to maximize their profits from selling cookies?

Part D What is the maximum profit?