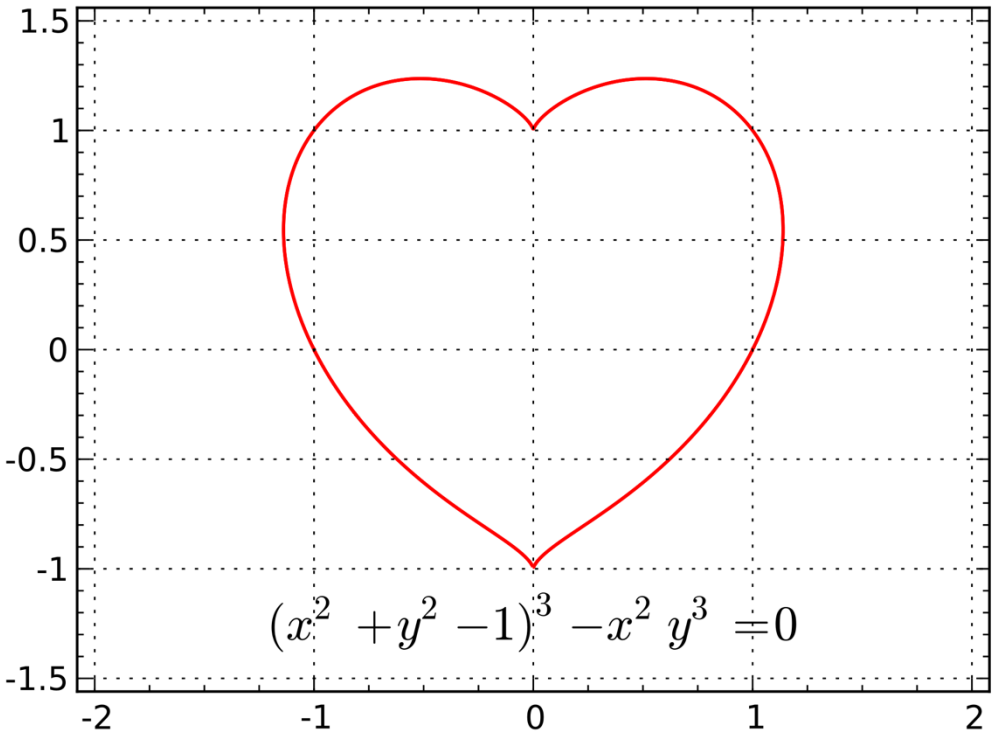
Heart of Algebra



Solving Linear Equations and Linear Inequalities

Basic Example

Which of the following best describes the solutions to inequality shown below:

$$3l - 6 \geq 8$$

(A) $l \geq \frac{2}{3}$

(B) $l \geq 2$

(C) $l \geq \frac{14}{3}$

(D) $l \geq 14$

Hard Example

In the equation shown below, *a* is constant. For what value of *a* does the equation have infinitely many solutions?

$$3 + 10x - 5 = (a + 1) \cdot x - 2$$

(A) 2

(B) 7

(C) 10

(D) 9

Interpreting Linear Functions

Basic Example

The amount of money that farmers in Alabama paid to maintain their crops between 1991 and 2008 is modeled by the equation below, where *P* is the amount of money the farmers paid, in millions of dollars, and *t* is the year (assuming 1991 is *t* = 0), what does the 3.53 mean in the equations?

$$P = 3.53t + 100$$

(A) The cost for maintaining crops was $3.53 million in 1991

(B) The cost for maintain crops was #3.53 million in 2008

(C) The costs for maintaining crops increased a total of $3.53 million between 1991 and 2008

(D) The costs for maintaining crops increased by $3.53 million each year between 1991 and 2008

Hard Example

Alice fills up the gas tank of her car before going for a long drive. The equation below models the amount of gas, *g,* in gallons, in Alice's car when she has driven *m* miles. What is the meaning of 32 in the equation?

$$g = 15 - \frac{m}{32}$$

(A) Alice uses 32 gallons of gas per mile

(B) Alice's tank can hold 32 gallons of gas

(C) Alice can drive 32 miles on a tank of gas

(D) Alice's car can travel 32 miles to the gallon

Linear Equation Word Problems

Basic Example

Kaylee wants to do well in her classes, so she is budgeting her time carefully to decide the number of classes, *c,* she will take this year. For each class that she takes, she expects to spend $2\frac{1}{2}$ hours each week working on homework. She expects to spend an additional $6\frac{1}{2}$ hours each week completing the assigned reading for all of her classes together. If Kaylee has 19 hours available each week to complete homework and reading for her classes, which equation best models the situation?

(A) $2\frac{1}{2}c - 6\frac{1}{2} = 19$

(B) $2\frac{1}{2}c + 6\frac{1}{2} = 19$

(C) $6\frac{1}{2}c - 2\frac{1}{2} = 19$

(D) $6\frac{1}{2}c + 2\frac{1}{2} = 19$

Linear Inequality Word Problems

Basic Example

A convenience store requires that Ayumi spend $4 or more if she wants to pay using a debit card. Donuts cost $0.80 each. A bottle of orange juice costs $1.20. If *d* represents the number of donuts Ayumi would need to buy to pay for 1 orange juice and the donuts using a debit card, which of the following inequalities best models the situation described above?

(A) $0.8(d + 1.2) > 4$

(B) $0.8(d + 1.2) \geq 4$

(C) $0.8d + 1.2 > 4$

(D) $0.8d + 1.2 \geq 4$

Harder Example

Gail needs to send out 300 wedding invitations. In 1 minute, she can put 6 invitations into envelopes and apply stamps to them. It takes her a minimum of 50 seconds to address each invitation by hand. If n represents the number of invitations Gail can prepare for mailing in 180 minutes, which of the following inequalities best models the situation above.

(A) $180 \geq \frac{n}{6} + \frac{5n}{6}$

(B) $180 > \frac{n}{6} + \frac{5n}{6}$

(C) $300 \leq 6n + \frac{6}{5}n$

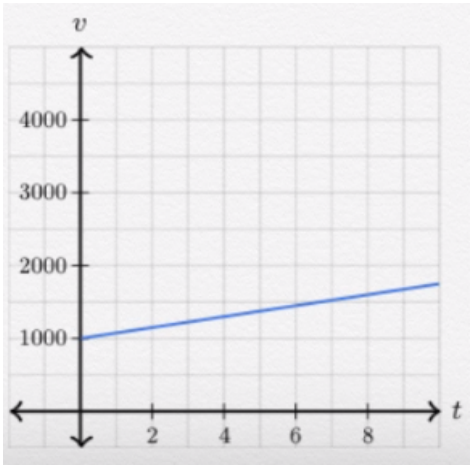
(D) $300 < 6n + \frac{6}{5}$

Graphing Linear Equations

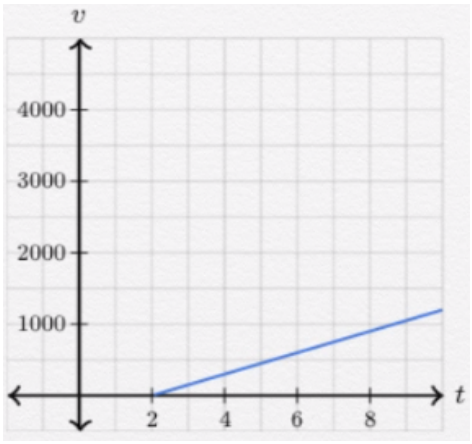
Basic Example

The value of a bond on January 1st, 2014 is $1,000. Each year the value of the bond increases linearly by $75. Which graph below represents *v,* the dollar value of the bond, as a function of *t,* the number of years after January 1st, 2014?

(A)



(B)



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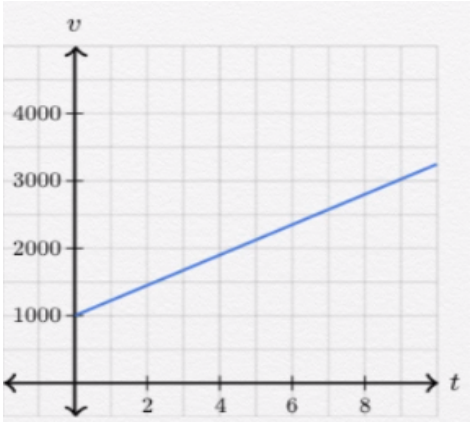
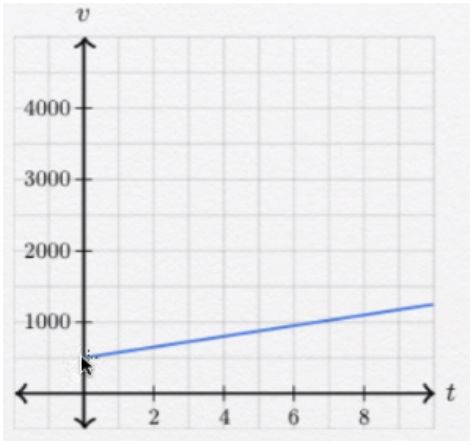
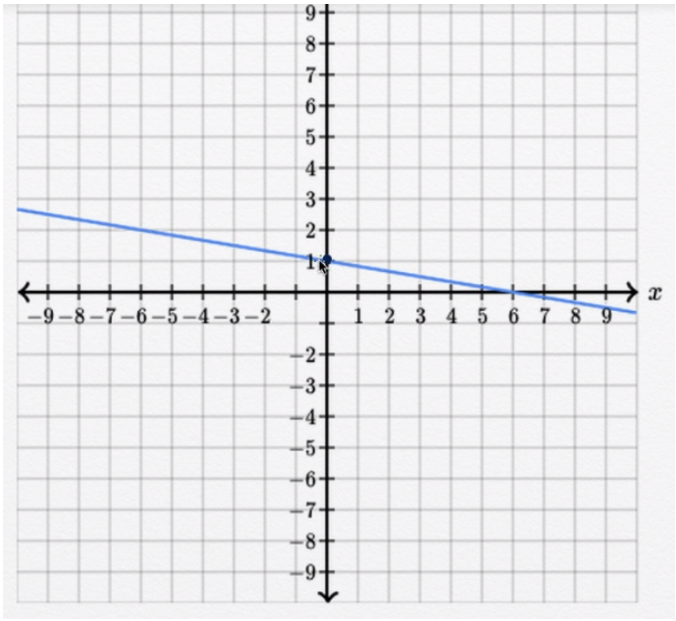


(D)



Harder Example

A line is graphed in the xy-plane as shown below. Which of the following equations represents the line?



(A) $x + 6y = 1$

(B) $x + 6y = 6$

(C) $x - 6y = 1$

(D) $x - 6y = -6$

Linear Function Word Problems

Basic Example

A college bookstore charges $60 for a yearly membership. The first book is free with the membership, and any book after that costs $7.60 including tax. How much money, *m,* does a student spend after buying *b* books and a yearly membership?

(A) $m = 7.60b$

(B) $m = 7.60(b - 1)$

(C) $m = 7.60b + 60$

(D) $m = 7.60(b - 1) + 60$

Harder Example

Abby's house is located 1.4 miles from her school. When she walks home from school, it takes her an average of 24 minutes. Assuming that Abby walks at a constant rate, which of the following functions best models Abby's distance from home, *d,* in miles, if she has walked a total of *t* minutes on her trip home that day?

(A) $d = 1.4 - \frac{7}{120}t$

(B) $d = 1.4 - 24t$

(C) $d = 1.4 - \frac{120}{7}t$

(D) $d = 1.4 + \frac{7}{120}t$

Systems of Linear Inequalities Word Problems

Basic Example

Brandon is a gym owner who wants to offer schedule of yoga and circuit training classes. Yoga classes are each 1.5 hours long, while circuit training classes are only 1 hour, and he wants at least 25 hours of classes on the schedule each week. All of his instructors are paid $35 per class, but Brandon doesn't want to spend more than $1,000 per week on salaries. Which of the following falls within Brandon's guidelines for the weekly schedule?

(A) 3 yoga classes and 19 circuit training classes

(B) 10 yoga classes and 12 circuit training classes

(C) 20 yoga classes and 12 circuit training classes

(D) 25 yoga classes and 6 circuit training classes

Harder Example

A cell phone producer distributes boxes of units to retail stores. A unit is either a cell phone or an accessory, and each box can have up to 24 units composed of c cellphones and a accessories. In addition, each box must have at least as many cell phones as accessories. Which of the following systems of inequalities best models the situation described above?

(A) $\begin{cases} 24 \leq a + c \\ a \leq c \end{cases}$

(B) $\begin{cases} 24 \leq a + c \\ c \leq a \end{cases}$

(C) $\begin{cases} a + c \leq 24 \\ a \leq c \end{cases}$

(D) $\begin{cases} a + c \leq 24 \\ c \leq a \end{cases}$

Solving Systems of Linear Equations

Basic Example

Consider the system of equations below. How many solutions does this system have?

$$y = 3x$$
$$x = 3y$$

(A) 0

(B) 1

(C) 2

(D) Infinitely Many

Harder Example

Consider the system of linear equations above. Which of the following choices of a will result in a system of equations with no solutions?

$$9x - 14y = -3$$
$$2x - ay = -6$$

(A) $-\frac{9}{14}$

(B) $\frac{-28}{9}$

(C) $\frac{9}{14}$

(D) $\frac{28}{9}$

Harder Example

Tickets for a play were \$2 for each child and \$4 for each adult. At one showing of the play, one adult brought 4 children and the remaining adults brought 2 children each. The total ticket sales from the children and adults was \$60 How many children and adults attended the play?

(A) 12 children and 9 adults
(B) 14 children and 8 adults
(C) 16 children and 7 adults
(D) 18 children and 6 adults

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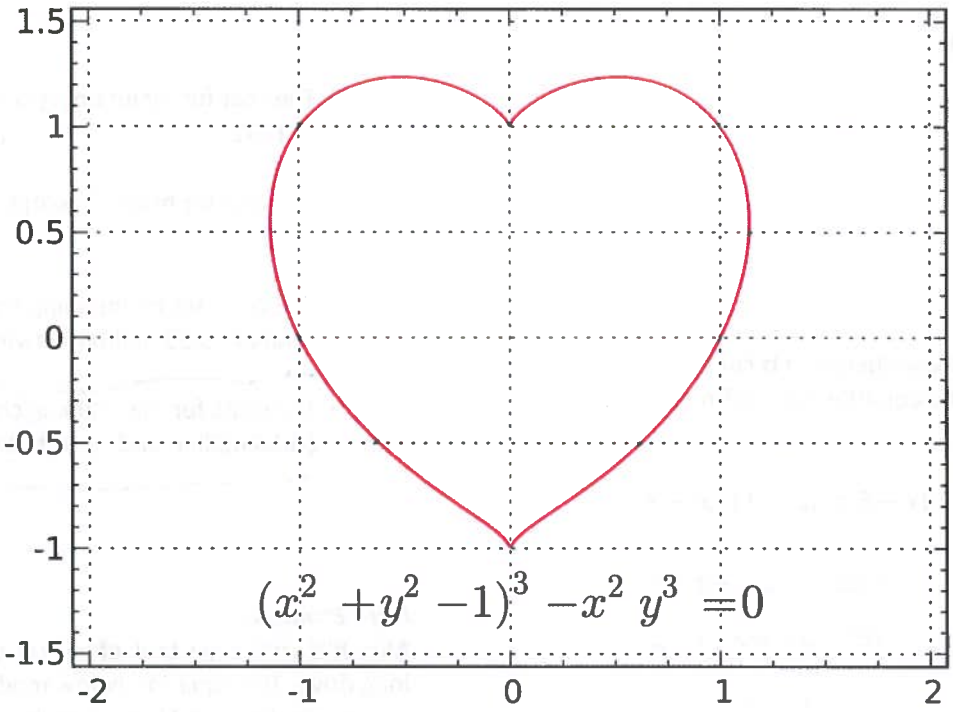
Heart of Algebra



Solving Linear Equations and Linear Inequalities

Basic Example

Which of the following best describes the solutions to inequality shown below:

$$3l - 6 \geq 8$$

(A) $l \geq \frac{2}{3}$

(B) $l \geq 2$

(C) $l \geq \frac{14}{3}$

(D) $l \geq 14$

$3l \geq 14$

$l \geq \frac{14}{3}$

Hard Example

In the equation shown below, a is constant. For what value of a does the equation have infinitely many solutions?

$$3 + 10x - 5 = (a + 1) \cdot x - 2$$

(A) 2

(B) 7

(C) 10

(D) 9

$3+10x-5=ax+x-2$

$10x-2=ax+x-2$

$10x=ax+x$

$9x=ax$

$9=a$

$a=9$

Interpreting Linear Functions

Basic Example

The amount of money that farmers in Alabama paid to maintain their crops between 1991 and 2008 is modeled by the equation below, where P is the amount of money the farmers paid, in millions of dollars, and t is the year (assuming 1991 is $t = 0$), what does the 3.53 mean in the equations?

$$P = 3.53t + 100$$

(A) The cost for maintaining crops was $3.53 million in 1991

(B) The cost for maintain crops was #3.53 million in 2008

(C) The costs for maintaining crops increased a total of $3.53 million between 1991 and 2008

(D) The costs for maintaining crops increased by $3.53 million each year between 1991 and 2008

Hard Example

Alice fills up the gas tank of her car before going for a long drive. The equation below models the amount of gas, g, in gallons, in Alice's car when she has driven m miles. What is the meaning of 32 in the equation?

$$g = 15 - \frac{m}{32}$$

$g = 15\,gal - \frac{miles}{32\,\frac{miles}{gal}} \rightarrow g = 15\,gal - miles \cdot \frac{gal}{32\,miles}$

(A) Alice uses 32 gallons of gas per mile

(B) Alice's tank can hold 32 gallons of gas

(C) Alice can drive 32 miles on a tank of gas

(D) Alice's car can travel 32 miles to the gallon

You are only left with gallons so we can conclude it must be miles/gallon

Linear Equation Word Problems

Basic Example

Kaylee wants to do well in her classes, so she is budgeting her time carefully to decide the number of classes, c, she will take this year. For each class that she takes, she expects to spend $2\frac{1}{2}$ hours each week working on homework. She expects to spend an additional $6\frac{1}{2}$ hours each week completing the assigned reading for all of her classes together. If Kaylee has 19 hours available each week to complete homework and reading for her classes, which equation best models the situation?

(A) $2\frac{1}{2}c - 6\frac{1}{2} = 19$

(B) $2\frac{1}{2}c + 6\frac{1}{2} = 19$

(C) $6\frac{1}{2}c - 2\frac{1}{2} = 19$

(D) $6\frac{1}{2}c + 2\frac{1}{2} = 19$

Linear Inequality Word Problems

Basic Example

A convenience store requires that Ayumi spend \$4 or more if she wants to pay using a debit card. Donuts cost \$0.80 each. A bottle of orange juice costs \$1.20. If d represents the number of donuts Ayumi would need to buy to pay for 1 orange juice and the donuts using a debit card, which of the following inequalities best models the situation described above?

(A) $0.8(d + 1.2) > 4$

(B) $0.8(d + 1.2) \geq 4$

(C) $0.8d + 1.2 > 4$

(D) $0.8d + 1.2 \geq 4$

Harder Example

Gail needs to send out 300 wedding invitations. In 1 minute, she can put 6 invitations into envelopes and apply stamps to them. It takes her a minimum of 50 seconds to address each invitation by hand. If n represents the number of invitations Gail can prepare for mailing in 180 minutes, which of the following inequalities best models the situation above.

∴ means Therefore

6 invitations/minute

∴ It takes her 10 seconds per invitation

$\frac{10 \text{ sec}}{60 \text{ sec}} = \frac{1}{6}$

$\frac{50 \text{ sec}}{60 \text{ sec}} = \frac{5}{6}$

$180 \geq n\left(\frac{1}{6} + \frac{5}{6}\right)$

(A) $180 \geq \frac{n}{6} + \frac{5n}{6}$

(B) $180 > \frac{n}{6} + \frac{5n}{6}$

(C) $300 \leq 6n + \frac{6}{5}n$

(D) $300 < 6n + \frac{6}{5}$

Graphing Linear Equations

Basic Example

The value of a bond on January 1st, 2014 is $1,000. Each year the value of the bond increases linearly by $75. Which graph below represents *v*, the dollar value of the bond, as a function of *t*, the number of years after January 1st, 2014?

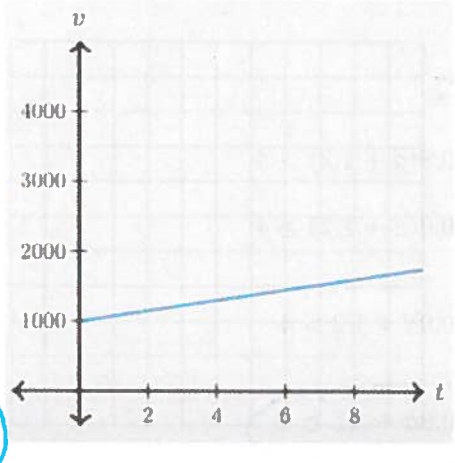
t = # of years
v = dollar value of bond

when t=0, you must be at $1,000

$t(8) = 8(\$75) = \$600$

∴ @ t=8, bond should be at $1600

(A)



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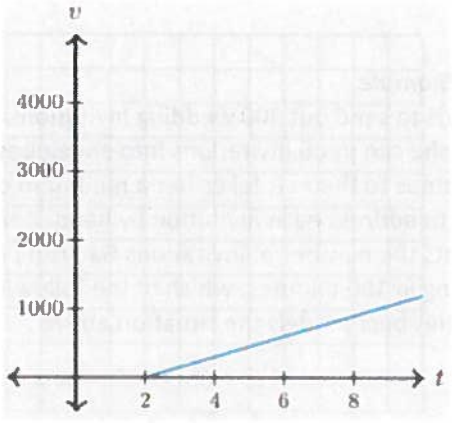

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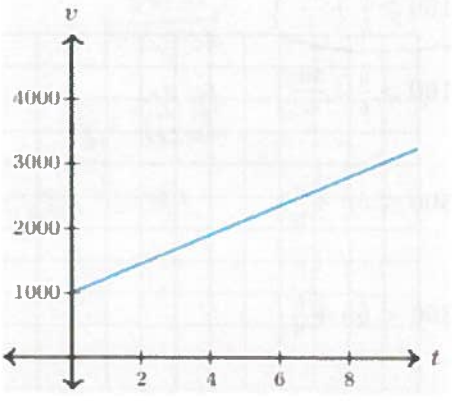

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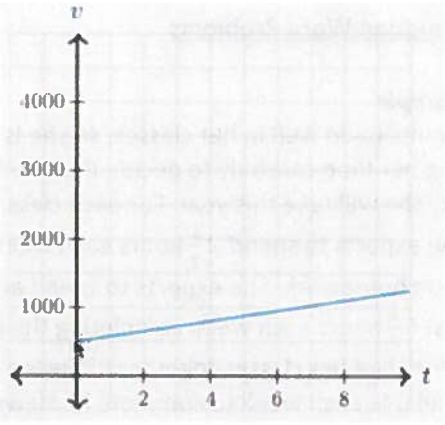
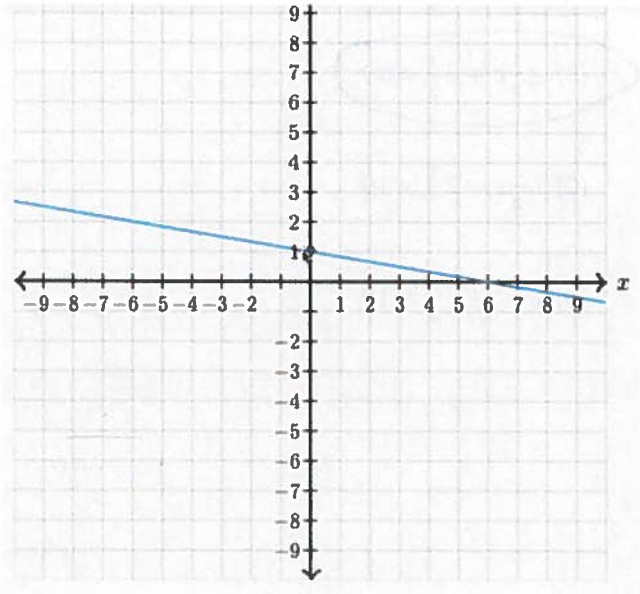


Harder Example

A line is graphed in the xy-plane as shown below. Which of the following equations represents the line?



(A) $x + 6y = 1$

(B) $x + 6y = 6$

(C) $x - 6y = 1$

(D) $x - 6y = -6$

y-int = 1

$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-1}{6}$

$y = mx + b$

$6\left[y = -\frac{1}{6}x + 1\right]$

$6y = -x + 6$

$x + 6y = 6$

Linear Function Word Problems

Basic Example

A college bookstore charges $60 for a yearly membership. The first book is free with the membership, and any book after that costs $7.60 including tax. How much money, *m*, does a student spend after buying *b* books and a yearly membership?

b/c means because

m = $ student spends
b = # of books

m = 7.6(b-1) + 60
↳ b/c the first book is free!

(A) $m = 7.60b$

(B) $m = 7.60(b-1)$

(C) $m = 7.60b + 60$

(D) $m = 7.60(b-1) + 60$

Harder Example

Abby's house is located 1.4 miles from her school. When she walks home from school, it takes her an average of 24 minutes. Assuming that Abby walks at a constant rate, which of the following functions best models Abby's distance from home, *d*, in miles, if she has walked a total of *t* minutes on her trip home that day?

24 minutes on avg.
d = distance from home

(A) $d = 1.4 - \frac{7}{120}t$

(B) $d = 1.4 - 24t$

(C) $d = 1.4 - \frac{120}{7}t$

(D) $d = 1.4 + \frac{7}{120}t$

Rate of walking = $\frac{1.4 \text{ miles}}{24 \text{ minutes}}$

Multiply by 10 to get rid of decimal = $\frac{14 \text{ miles}}{240 \text{ minutes}}$

$= \frac{7 \text{ miles}}{120 \text{ minutes}}$

$\therefore d = 1.4 - \frac{7}{120}t$

Systems of Linear Inequalities Word Problems

Basic Example

Brandon is a gym owner who wants to offer schedule of yoga and circuit training classes. Yoga classes are each 1.5 hours long, while circuit training classes are only 1 hour, and he wants at least 25 hours of classes on the schedule each week. All of his instructors are paid $35 per class, but Brandon doesn't want to spend more than $1,000 per week on salaries. Which of the following falls within Brandon's guidelines for the weekly schedule?

yoga = 1.5 hours
circuit training = 1 hour

Must be < $1,000 & more than 25 hours of classes

(A) 3 yoga classes and 19 circuit training classes

(B) 10 yoga classes and 12 circuit training classes

(C) 20 yoga classes and 12 circuit training classes

(D) 25 yoga classes and 6 circuit training classes

10(1.5 hours) + 12(1 hour) = 27 hours ✓
$35 × 22 = $770 ✓

Harder Example

A cell phone producer distributes boxes of units to retail stores. A unit is either a cell phone or an accessory, and each box can have up to 24 units composed of *c* cellphones and *a* accessories. In addition, each box must have at least as many cell phones as accessories. Which of the following systems of inequalities best models the situation described above?

c = cellphones
a = accessories

have up to $24 \le a + c$

$c \ge a$

(A) $\begin{cases} 24 \le a + c \\ a \le c \end{cases}$

(B) $\begin{cases} 24 \le a + c \\ c \le a \end{cases}$

(C) $\begin{cases} a + c \le 24 \\ a \le c \end{cases}$

(D) $\begin{cases} a + c \le 24 \\ c \le a \end{cases}$

Solving Systems of Linear Equations

Basic Example

Consider the system of equations below. How many solutions does this system have?

$$y = 3x$$
$$x = 3y$$

(A) 0

(B) 1

(C) 2

(D) Infinitely Many

$y = 3x$

Substitution!

$x = 3(3x)$

$x = 9x$

$0 = 8x$

$x = 0$ $\quad (0,0)$

$y = 3(0)$

$y = 0$

Therefore, they meet at point $(0,0)$ & have 1 solution

Harder Example

Consider the system of linear equations above. Which of the following choices of a will result in a system of equations with no solutions?

① $9x - 14y = -3$

② $2x - ay = -6$

$y = mx + b$ (slope, y-int)

// lines have no solutions

∴ they must have same slope

(A) $-\frac{9}{14}$

(B) $\frac{-28}{9}$

(C) $\frac{9}{14}$

(D) $\frac{28}{9}$

① $-14y = -9x - 3$

$y = \frac{9}{14}x + \frac{3}{14}$

② $-ay = -2x - 6$

$y = \frac{2}{a}x + \frac{6}{a}$

$\therefore \frac{9}{14} = \frac{2}{a}$

$9a = 28$

$a = \frac{28}{9}$

Harder Example

Tickets for a play were \$2 for each child and \$4 for each adult. At one showing of the play, one adult brought 4 children and the remaining adults brought 2 children each. The total ticket sales from the children and adults was \$60 How many children and adults attended the play?

(A) 12 children and 9 adults

(B) 14 children and 8 adults

(C) 16 children and 7 adults

(D) 18 children and 6 adults

$A = \$4$

$2C = \$4$

A
C C
C C

A
C C
C C
etc

$A = \$4$

$4C = 4(\$2) = \$8$

$\$12$

$A = \$4$

$2C = 2(\$2) = \$4$

$\$8$

$\$60 = 12 + 8r$

$\$48 = 8r$

$r = 6$

∴ 6 adults w/ 2 children

Don't forget about the FIRST adult!

$1 + 6 = 7$